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U. S. DEPARTMENT OF AGRICULTURE,
WEATHER BUREAU.

STUDIES ON THE THERMODYNAMICS OF THE
ATMOSPHERE.

Reprints from the Monthly Weather Review, January, February, March, June, July, August, October, November,
and December, 1906.

BY

FRANK HAGAR BIGELOW, M. A., L. H. D.,
Professor of Meteorology.

Prepared under the direction of WILLIS L. MOORE, Chief U. S. Weather Bureau.

WASHINGTON:
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From the right spherical triangle PBE , figs. 2 and 3, we have—

$$(1) \quad \cos \delta = \cos \alpha \cos \beta.$$

$$(2) \quad \cot H. A. = \cot \alpha \sin \beta.$$

Let α and β be two variables in Cartesian rectangular coordinates. Assign δ a constant value and let α and β vary; then from the first of these equations a certain definite curve will be traced as shown in the diagram (see fig. 4), and by assigning arbitrary values to δ , from 0° up to 90° , a family of declination curves will be obtained. In the same manner there will result a family of hour angle curves (see fig. 5) by assigning arbitrary values to the hour angle $H. A.$, and then letting α and β vary. By constructing the two series of curves on the same set of coordinate axes these results follow.

Thus the plane area is divided into a series of curve line quadrilaterals which may be made as small as we please by tracing the curves at sufficiently small intervals.

If one value of δ and one of $H. A.$ be given, then by means of these curves the position of a point is fixed in the plane. The rectangular coordinates of this point are the values of α and β corresponding to these values of δ and $H. A.$ Conversely, if we know the values of α and β , we are enabled to plot the position in the plane, and we can read off δ and $H. A.$ by means of the curves.

By means of this abacus, the azimuth and altitude of a star may be determined when its declination and hour angle and the estimated colatitude are given. Plot the position of the star by means of δ and $H. A.$ curves, estimating the minutes by the eye, and read the rectangular coordinates of the point thus plotted, α on the vertical scale and β on the horizontal scale. The $H. A.$ curves here represent meridians and the δ curves represent parallels of the celestial sphere. Now make $B = \lambda + \beta$, considering β negative if the latitude and declination are of contrary name. Plot the point which has α and B for rectangular coordinates, and, considering now the $H. A.$ curves to represent verticals, and the δ curves to represent circles of equal latitude, read off the azimuth by means of the $H. A.$ curves, and the altitude due to the estimated latitude by means of the δ curves.

This abacus will also serve to find:

(1) The time of rising and setting of a star and its azimuth in the horizon.

(2) The name of an observed star.

(3) The distance and great circle course between two points.

Through the initiative of Monsieur Engène Pereire, President of the Administrative Council of the Compagnie Générale Transatlantique, this method has been published in rectangular coordinates on four grand-eagle pages on a scale of $\frac{1}{10000}$ of a meter to the degree, and may be purchased in France.

STUDIES ON THE THERMODYNAMICS OF THE ATMOSPHERE.

By Prof. FRANK H. BIGELOW.

I.—ASYMMETRIC CYCLONES AND ANTICYCLONES IN EUROPE AND AMERICA.

INTRODUCTORY REMARKS.

The synthetic construction of the correct statement of the mechanical problems involved in the cyclonic and anticyclonic circulations of the atmosphere depends upon the coordination of the data derived from observations of the velocity vectors, the pressures, and the temperatures prevailing in the moving air masses. In my International Cloud Report, 1898, and in the MONTHLY WEATHER REVIEWS for January, February, and March, 1902, the distribution of the velocity vectors for the United States was described; in the REVIEWS for January and February, 1903, the distribution of the pressures corresponding to these velocities was explained; in this present series of papers the results of my studies on the relation of the tem-

peratures to the velocities and pressures will be summarized. It has been shown conclusively for the United States that there are no true local warm-centered and cold-centered cyclones or anticyclones in the atmosphere, and that all the theoretical discussions or theses founded on that basis are misdirected. The observations demonstrate that in the lower atmosphere the actual mechanism consists of rather deep warm and cold countercurrents of air, which underrun the prevailing eastward drift. The centers of gyration are uniformly in the region where these counterflowing currents meet each other, that is to say, on the edges rather than in the midst of the warm and cold regions. About one half of the cyclone is relatively warm and the other half cold, while the opposite half of the anticyclone is warm and its alternate half is cold. Thus, in the United States, the eastern and northern sectors of the cyclone with the western and northern sectors of the anticyclone are warm, while the western and southern sectors of the cyclone and the eastern and southern sectors of the anticyclone are cold. The warm air flowing from the southwest into the east of the cyclone and west of the anticyclone, and the cold air flowing from the northwest into the east of the anticyclone and the west of the cyclone, constitute two currents whose temperatures differ from each other and from the normal temperature of the prevailing eastward drift. These currents seek to equalize their different temperatures by interpenetration, and in so doing the circulating structures known as cyclonic and anticyclonic are established. The heat added to the tropical zones of the earth by the solar radiation is to a considerable extent transported into the temperate zones by long horizontal currents in the lower levels, and is there expended in generating local circulations. These penetrate the upper current of eastward drift and tend to retard its motion, slowing it down to the moderate velocities which have been found to exist within ten miles of the ground. This stratification and interpenetration of currents of different temperatures is the true source of the energy of storms. The heat energy derived from the condensation of aqueous vapor to water, and the energy produced by purely dynamic eddies are entirely secondary in importance to the thermodynamic energy obtained by the counterflow and underflow of warm southerly currents against the cold northerly currents and beneath the eastward flowing drift. In the present series of papers it is purposed to examine somewhat fully the thermodynamic conditions which exist in the atmosphere, especially in cyclones, anticyclones and tornadoes; at present the temperature data are inadequate for a satisfactory consideration of hurricanes and the general circulation, though something may also be done in that direction.

THE SUPPOSED DIFFERENCE IN THE TEMPERATURE DISTRIBUTION BETWEEN AMERICAN AND EUROPEAN CYCLONES AND ANTICYCLONES.

Certain discussions of the available temperature observations made at different levels in cyclones and anticyclones for the United States and Europe indicate that there is a serious disagreement in the results for the respective regions, as if these local circulations might really be different in some important respects. We should not expect to find any such divergence in the thermodynamics of the atmosphere when the observations and the computations have been accurately made, but as it is comparatively difficult to extract the exact truth of the matter from the actual observations, it will be proper to examine these observations carefully before admitting that any important difference in the structures actually exists. A suitable review of the literature may be found in Mr. Clayton's article,¹ from which the following few statements are compiled:

From a study of mountain observations, Professor Hann found

¹ Various researches on the temperatures in cyclones and anticyclones in temperate latitudes. By H. Helm Clayton. Beiträge zur Physik der freien Atmosphäre. Vol. I, pp. 93-106.

TABLE 1.—*Temperature-falls at Blue Hill.*
HIGH AREAS IN WINTER (OCTOBER TO MARCH).

Height in meters.	North (3).				East (9).				South (10).				West (3).				Mean.	
	F	No.	ΔT	T	F	No.	ΔT	T	F	No.	ΔT	T	F	No.	ΔT	T	T	$\frac{\Delta T}{100}$
	$^{\circ} F.$		$^{\circ} F.$	$^{\circ} C.$	$^{\circ} F.$		$^{\circ} F.$	$^{\circ} C.$	$^{\circ} F.$		$^{\circ} F.$	$^{\circ} C.$	$^{\circ} F.$		$^{\circ} F.$	$^{\circ} C.$	$^{\circ} C.$	$^{\circ} C.$
4000		..	-21.2	-7.9		..	-37.4	-18.8		..	-31.0	-17.9		..	-29.8	-7.6	-13.1	-0.50
3800	-10.5	1	-19.8	-7.3	-36.1	2	-35.2	-17.6		..	-28.9	-16.8		..	-28.0	-6.6	-12.1	-0.55
3600	-18.7	3	-18.2	-6.4	-32.0	2	-33.2	-16.8		..	-27.0	-15.7		..	-26.6	-5.8	-11.1	-0.50
3400	-17.5	2	-16.8	-6.6	-32.8	3	-30.9	-15.1	-20.5	1	-25.0	-14.6		..	-25.2	-5.0	-10.1	-0.50
3200	-13.9	2	-15.2	-4.7	-27.9	4	-28.7	-14.0	-20.6	1	-23.1	-18.6		..	-23.8	-4.2	-9.1	-0.50
3000	-13.7	2	-13.7	-3.9	-32.1	5	-26.6	-12.7	-17.8	2	-21.3	-12.6		..	-21.8	-3.1	-8.1	-0.45
2800	-12.5	3	-12.2	-3.1	-23.6	8	-24.6	-11.6	-19.2	3	-19.8	-11.7		..	-20.5	-2.4	-7.2	-0.40
2600	-10.2	3	-11.0	-2.4	-15.0	5	-22.3	-10.4		..	-18.4	-10.9	-12.7	1	-19.0	-1.6	-6.4	-0.40
2400	-10.1	3	-10.2	-2.0	-20.8	1	-20.4	-9.3		..	-16.7	-10.0	-17.7	1	-17.7	-0.9	-5.6	-0.40
2200	-6.4	1	-9.4	-1.5	-16.3	5	-18.6	-8.3	-17.4	1	-15.8	-9.4	-13.2	2	-16.1	0.0	-4.8	-0.45
2000	-6.9	5	-8.8	-1.2	-18.3	7	-16.5	-7.1	+ 6.5	1	-13.7	-8.3		..	-14.4	1.0	-3.9	-0.60
1800	-9.2	4	-6.0	0.4	-12.5	6	-14.7	-6.2		..	-11.6	-7.2	-17.6	1	-12.4	2.1	-2.7	-0.50
1600	-8.2	4	-4.2	1.4	-16.8	4	-12.6	-5.0	+ 2.3	5	-9.4	-5.9	-12.0	2	-10.4	3.4	-1.5	-0.40
1400	+ 0.2	1	-3.0	2.0	-10.9	13	-11.3	-4.3	-4.5	8	-6.8	-4.5	-8.5	3	-9.2	3.9	-0.7	-0.20
1200	-5.7	3	-2.6	2.1	-12.0	8	-10.4	-3.8	-6.8	6	-5.3	-3.7	-7.4	7	-8.5	4.2	-0.3	-0.05
1000	+ 0.8	2	-3.5	1.8	-8.0	11	-9.8	-3.5	-2.5	8	-5.2	-3.6	-7.6	5	-8.2	4.5	-0.2	-0.16
800	-3.1	4	-4.5	1.1	-5.0	18	-8.7	-2.8	-5.5	13	-5.8	-3.9	-9.5	7	-7.2	5.0	0.1	-0.10
600	-7.9	1	-4.4	1.3	-9.3	15	-6.8	-1.8	-7.8	17	-5.0	-4.0	-6.4	6	-6.0	5.7	0.3	-0.45
400	-3.7	4	-3.6	1.7	-4.1	16	-4.3	-0.4	-5.0	16	-5.2	-3.6	-3.3	1	-3.7	7.0	1.2	-0.55
200	-1.3	50	-1.8	2.7	-1.4	151	-2.0	0.9	-2.5	107	-3.2	-2.5	-2.1	36	-2.0	7.9	2.3	-0.60
0	38.7	50	38.7	3.7	35.5	161	35.5	2.0	30.7	107	30.7	-0.7	48.2	36	48.2	9.0	3.5	
																	Mean	-0.42

LOW AREAS IN WINTER.

Height in meters.	North (0).				East (5).				South (13).				West (7).				Mean.	
	F	No.	ΔT	T	F	No.	ΔT	T	F	No.	ΔT	T	F	No.	ΔT	T	T	$\frac{\Delta T}{100}$
	$^{\circ} F.$		$^{\circ} F.$	$^{\circ} C.$	$^{\circ} F.$		$^{\circ} F.$	$^{\circ} C.$	$^{\circ} F.$		$^{\circ} F.$	$^{\circ} C.$	$^{\circ} F.$		$^{\circ} F.$	$^{\circ} C.$	$^{\circ} C.$	$^{\circ} C.$
4000		..				..	-23.7	-13.2		..	-45.0	-18.7		..	-24.0	-16.0	-15.0	-0.55
3800		..				..	-25.7	-12.0		..	-43.0	-17.6		..	-22.4	-15.1	-14.9	-0.45
3600		..				..	-25.2	-11.2		..	-40.8	-16.4		..	-20.9	-14.3	-14.0	-0.50
3400		..				..	-23.4	-10.2		..	-38.7	-15.2		..	-19.4	-13.5	-13.0	-0.55
3200		..				..	-21.4	-9.1	-37.9	2	-36.4	-13.9		..	-18.0	-12.7	-11.9	-0.55
3000		..				..	-19.7	-8.1	-35.2	2	-34.0	-12.6	-4.9	1	-16.0	-11.6	-10.8	-0.55
2800		..				..	-17.8	-7.1	-31.1	2	-32.0	-11.5	-3.2	3	-14.2	-10.6	-9.7	-0.50
2600		..				..	-15.8	-6.0	-30.1	3	-29.6	-10.2	+ 1.1	3	-12.8	-9.8	-8.7	-0.60
2400		..				..	-14.0	-5.0	-25.5	4	-27.0	-8.7	+ 3.5	2	-11.2	-8.9	-7.5	-0.45
2200		..			-10.9	3	-12.0	-3.9	-23.3	5	-24.8	-7.5	-9.6	4	-10.0	-8.3	-6.6	-0.45
2000		..			-8.3	6	-10.3	-3.0	-23.9	6	-22.7	-6.3	-4.5	5	-9.2	-7.8	-5.7	-0.40
1800		..			-6.5	3	-9.2	-2.3	-20.2	5	-20.3	-5.0	-10.7	7	-8.5	-7.4	-4.9	-0.20
1600		..			-10.7	3	-9.3	-2.4	-18.0	20	-18.0	-8.7	-9.5	5	-8.4	-7.3	-4.5	-0.15
1400		..			-8.5	7	-9.8	-2.7	-15.7	23	-16.0	-2.6	-7.2	7	-8.4	-7.3	-4.2	-0.25
1200		..			-10.2	8	-10.0	-2.8	-14.0	31	-13.8	-1.4	-8.2	8	-7.9	-7.0	-3.7	-0.25
1000		..			-10.0	5	-9.8	-2.7	-10.3	22	-11.4	-0.1	-9.6	8	-7.3	-6.8	-3.2	-0.40
800		..			-11.1	4	-8.9	-2.2	-8.2	17	-9.2	1.2	-4.8	7	-6.1	-6.1	-2.4	-0.40
600		..			-6.5	8	-7.5	-1.4	-8.0	21	-7.3	2.2	-5.5	11	-5.0	-5.5	-1.6	-0.55
400		..			-5.7	8	-5.9	-0.5	-4.1	13	-4.7	3.7	-4.0	13	-3.7	-4.7	-0.5	-0.65
200		..			-2.2	62	-3.0	1.1	-1.6	185	-2.4	5.0	-1.9	109	-2.0	3.8	0.8	-0.65
0		..			37.1	62	37.1	2.8	43.4	185	43.3	6.3	27.1	123	27.1	-2.7	2.1	
																	Mean	-0.43

that in the higher levels the temperature is relatively low over cyclones, but high over anticyclones, and thence he supposed that this apparent inversion must be due to dynamic actions, as of eddies or driven whirls in the prevailing eastward drift. To somewhat the same effect the investigations of Dechevrens, Berson, and Teisserenc de Bort have come, namely, that "cyclones average colder than anticyclones below nine kilometers." On the other hand, the investigations of Harrington, Rotch, Hazen, Clayton, Shaw, and Dines lead to the conclusion that "cyclones average warmer than anticyclones."

The discussion of the temperatures over Hald and Berlin by Grenander² is said to support Hann's proposition rather than the American view. It is worth while to find whether this contradiction can be readily reconciled. Without examining critically the observations themselves and the methods of reduction that have been employed, it is sufficient to remark that a very large number of observations is required to arrive at definitive results, and that both the annual and the diurnal variations of temperature must be eliminated from the

temperature-falls from the surface. Mountain observations require a series of local corrections to be applied to reduce them to free air measures of temperature, and, generally, the observations must be distributed quite uniformly throughout the 24 hours. The tendency is to use an excess of daytime observations, and this will produce one-sided data in the strata up to at least the elevation of 3000 meters. In the following exposition it will be sufficient to employ the data contained in Grenander's paper and in the Blue Hill paper,³ which have already been used in my discussion of the diurnal periods of the temperature. We shall expect to show that the structures of the American and European cyclones and anticyclones are practically the same, except possibly in the lower strata, within 2000 meters of the surface.

THE TEMPERATURE-FALLS AT BLUE HILL AND HALD-BERLIN.

The reader is referred to my paper on "The diurnal periods of the temperature"⁴ for a description of the method employed in discussing the Blue Hill kite observations of 1897-1902.

² Observations at the Blue Hill Observatory, 1901-2, and appendix of the observations with kites, 1897-1902, with discussion by H. Helm Clayton.

⁴ Monthly Weather Review, February, 1905.

³ Les gradients verticaux de la température dans les minima et les maxima barométriques. Par S. Grenander. Arkiv för matematik, astronomi och fysik. Bd. 2, No. 7. (1905.)

TABLE 2.—*Temperature-falls at Blue Hill.*
HIGH AREAS IN SUMMER (APRIL TO SEPTEMBER).

Height in meters.	Center (1).				East (24).				South (12).				West (8).				Mean.	
	F	No.	ΔT	T	F	No.	ΔT	T	F	No.	ΔT	T	F	No.	ΔT	T	T	$\frac{\Delta T}{100}$
	° F.		° F.	° C.	° F.		° F.	° C.	° F.		° F.	° C.	° F.		° F.	° C.	° C.	° C.
4000		...	-37.4	-2.9	-48.3	1	-45.2	-8.4		...	-43.3	-9.6		...	-41.2	-4.8	-6.4	-0.80
3800	-38.9	2	-35.0	-1.5	-40.0	1	-42.1	-6.7		...	-40.0	-7.7		1	-38.7	-3.4	-4.8	-0.75
3600	-30.8	2	-32.6	-0.2	-45.2	1	-39.1	-5.1		...	-36.7	-5.9		1	-36.2	-2.0	-3.3	-0.75
3400	-31.8	4	-30.2	1.1	-33.2	1	-36.0	-3.3		1	-33.5	-4.1		2	-33.8	-0.7	-1.8	-0.75
3200	-27.2	2	-28.2	2.3		...	-33.0	-1.6		2	-30.2	-2.3		2	-31.5	0.6	-0.3	-0.70
3000	-27.7	5	-25.4	3.6	-32.6	7	-29.8	0.1		1	-27.2	-0.6		...	-29.0	2.0	1.1	-0.75
2800	-22.2	6	-23.3	4.9	-22.6	10	-27.3	1.5	-28.0	1	-24.6	0.8		...	-26.6	3.3	2.6	-0.70
2600	-21.6	6	-21.2	6.1	-22.1	5	-24.5	3.1	-17.1	1	-22.0	2.3		3	-24.4	4.6	4.0	-0.60
2400	-17.0	6	-19.4	7.1	-17.5	6	-22.2	4.4	-15.5	2	-20.0	3.4		5	-22.2	5.8	5.2	-0.60
2200	-16.0	5	-17.3	8.3	-20.7	9	-19.9	5.7	-7.7	3	-18.0	4.5		7	-20.0	7.0	6.4	-0.60
2000	-18.2	5	-15.4	9.4	-18.0	9	-17.4	7.1	-12.7	5	-15.9	5.8		8	-18.0	8.1	7.6	-0.60
1800	-14.9	11	-13.6	10.2	-18.7	15	-15.2	8.3	-15.2	6	-13.5	7.0		7	-16.0	9.2	8.7	-0.55
1600	-11.4	8	-11.8	11.3	-15.3	16	-12.6	9.7	-16.1	7	-11.2	8.3		8	-14.6	10.0	9.8	-0.55
1400	-10.2	11	-10.0	12.3	-11.4	20	-10.2	11.0	-7.6	15	-9.1	9.4		8	-13.5	10.6	10.3	-0.50
1200	-7.1	13	-8.5	13.2	-8.3	28	-7.4	12.5	-6.8	10	-7.2	10.5		9	-12.0	11.4	11.9	-0.30
1000	-7.1	16	-7.9	13.6	-4.7	35	-6.7	13.0	-5.7	17	-6.0	11.2		11	-10.7	12.1	12.5	-0.20
800	-7.9	17	-7.6	13.7	-2.2	38	-6.0	13.4	-4.8	14	-5.3	11.5		13	-9.2	13.0	12.9	-0.30
600	-1.9	24	-6.1	14.5	+2.6	46	-4.7	14.1	-5.4	19	-5.2	11.6		18	-7.8	13.8	13.5	-0.45
400	-4.3	25	-4.3	15.5	-1.9	46	-2.5	15.3	-5.3	19	-4.6	11.9		10	-6.2	14.7	14.4	-0.60
200	-1.1	178	-2.0	16.8	-0.9	326	-0.6	16.3	-2.6	134	-2.8	12.9		119	-2.7	16.6	15.7	-0.55
0	64.3	178	64.3	17.9	62.0	326	62.0	16.7	58.1	134	53.1	14.5		119	64.6	18.1	16.8	
																	Mean	-0.62

LOW AREAS IN SUMMER.

Height in meters.	Center (4).				East (5).				South (22).				West (7).				Mean.	
	F	No.	ΔT	T	F	No.	ΔT	T	F	No.	ΔT	T	F	No.	ΔT	T	T	$\frac{\Delta T}{100}$
	° F.		° F.	° C.	° F.		° F.	° C.	° F.		° F.	° C.	° F.		° F.	° C.	° C.	° C.
4000		...	-44.0	-9.8		...	-38.8	-6.0		...	-48.2	-3.6		...	-46.5	-9.0	-7.1	-0.60
3800		...	-42.3	-8.9		...	-36.4	-4.6		...	-45.0	-2.1		...	-44.8	-8.0	-5.9	-0.55
3600	-41.8	1	-40.4	-7.8	-38.8	2	-34.6	-3.6	-35.6	1	-43.2	-0.8		1	-43.0	-6.0	-4.8	-0.60
3400		...	-38.8	-7.0	-31.3	1	-32.2	-2.3	-37.9	5	-41.0	0.4		2	-40.7	-5.7	-3.6	-0.55
3200	-37.3	1	-37.0	-6.0	-33.3	2	-30.5	-1.4	-42.0	6	-38.8	1.6		4	-38.4	-4.4	-2.5	-0.60
3000	-35.5	3	-35.0	-4.8	-31.8	3	-28.6	-0.3	-31.9	13	-36.1	3.1		3	-36.4	-3.3	-1.3	-0.60
2800	-33.2	3	-33.2	-3.8	-35.5	4	-26.8	0.7	-35.2	11	-34.6	4.0		6	-34.0	-2.0	-0.3	-0.60
2600	-28.9	4	-31.0	-2.6	-29.1	2	-21.8	1.8	-30.9	14	-32.7	5.1		4	-31.5	-0.6	0.9	-0.60
2400	-29.7	6	-28.6	-1.3	-25.4	2	-23.2	2.7	-31.2	17	-30.4	6.3		9	-29.6	0.5	2.1	-0.65
2200	-26.8	7	-26.0	0.2	-21.5	3	-21.0	3.9	-28.8	18	-28.2	7.5		12	-27.2	1.8	3.4	-0.60
2000	-23.7	7	-23.5	1.6	-15.3	4	-19.4	4.8	-25.9	18	-25.8	8.9		9	-25.0	3.0	4.6	-0.63
1800	-25.3	3	-20.7	3.1	-15.4	4	-17.0	6.2	-24.0	23	-23.4	10.2		12	-23.2	4.0	5.9	-0.65
1600	-16.0	3	-18.0	4.6	-14.9	5	-15.1	7.2	-20.4	18	-20.8	11.6		9	-20.8	5.3	7.2	-0.65
1400	-14.1	6	-15.7	5.9	-17.6	5	-13.3	8.2	-18.2	25	-18.1	13.1		12	-18.2	6.8	8.5	-0.60
1200	-13.6	5	-13.2	7.3	-11.1	6	-11.6	9.1	-16.7	22	-15.6	14.5		11	-16.0	8.0	9.7	-0.55
1000	-12.1	6	-11.0	8.6	-5.5	3	-9.7	10.2	-14.2	25	-13.0	16.0		12	-14.6	8.8	10.8	-0.65
800	-9.7	3	-9.2	9.5	-10.1	3	-7.8	11.3	-11.0	33	-10.5	17.3		10	-12.0	10.2	12.1	-0.75
600	-7.4	5	-6.8	10.8	-5.7	8	-5.7	12.5	-8.9	38	-8.0	18.8		12	-7.8	12.3	13.6	-0.80
400	-3.4	8	-4.8	12.2	-5.8	11	-3.5	13.6	-5.5	40	-5.3	20.2		11	-4.0	14.7	15.2	-0.60
200	-3.4	73	-2.0	13.5	-2.7	82	-1.6	14.7	-2.7	341	-2.7	21.7		138	-2.0	15.8	16.4	-0.60
0	58.2	73	58.2	14.6	60.1	82	60.1	15.6	73.7	341	73.7	23.2		145	62.5	16.9	17.6	
																	Mean	-0.63

It is convenient to utilize the same data for discussing the distribution of temperature in the several strata of cyclones and anticyclones up to the height of about 4000 meters, for the sake of eliminating the diurnal variation. This was done by taking the mean values of the temperatures as observed at the several ascensions, which were distributed in the different hours of the day. While the distribution among the 24 hours is not completely uniform, it is probable that a fairly satisfactory elimination of the diurnal temperature variation in the levels from the surface to 3000 meters has been accomplished in my computations, by taking the average daily values as they exist in this report for the winter months (October to March) and the summer months (April to September), thus making two groups for the five years of observations. The number of available observations is not nearly great enough to give definitive mean values, but the results here explained probably indicate quite sufficiently the typical conditions that require to be known as the basis of a theoretical discussion of the thermodynamics of storms.

Table 1, temperature-falls at Blue Hill, winter high areas and low areas, and Table 2, summer high areas and low areas, summarize the data. The temperature-falls are given for the

N. E. S. W. sectors, and for the center when possible. These sectors group the data according to my subareas as follows: For center we take subareas (1, 2, 3, 4), for north (9, 10, 17, 18), east (7, 8, 15, 16), south (5, 6, 13, 14), west (11, 12, 19, 20). The areas at the time of observation at Blue Hill were located in respect to these subareas by studying the corresponding weather maps and selecting the most conspicuous high area or low area for the center.

EXPLANATION OF TABLES 1, 2, 3, AND 4.

The first column of Table 1 gives the height in meters, the second, *F*, the mean temperature-falls in Fahrenheit degrees, as determined by the number of observations recorded in column three; these temperature-falls were plotted on diagrams and average curves were drawn through the points, from which the values of ΔT found in column four were scaled, and these should represent more exact values than can be obtained from the limited number of observations at our disposal. At the bottom of columns *F* and ΔT is found the mean surface temperature, determined by the numerous readings made at the Blue Hill valley station. In column five, marked *T*, are given the corresponding temperatures themselves in centigrade degrees, being transformed from the Fahrenheit degrees re-

TABLE 3.—*Temperature-falls at Hald and Berlin.*

HIGH AREAS IN WINTER (OCTOBER TO MAY 15).

Height in meters.	North.		East.		South.		West.		Mean.	
	ΔT	T	ΔT	T	ΔT	T	ΔT	T	T	ΔT 100
	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$
6000	-31.6	-27.0	-34.0	-33.2	-25.3	-24.5	-25.7	-21.6	-26.6	-0.75
5800	-30.0	-25.4	-32.5	-31.7	-23.9	-23.1	-24.4	-20.3	-25.1	-0.70
5600	-28.3	-23.7	-31.0	-30.2	-22.7	-21.9	-22.9	-18.8	-23.7	-0.80
5400	-26.7	-22.1	-28.7	-27.9	-21.7	-20.9	-21.4	-17.3	-22.1	-0.80
5200	-25.5	-20.9	-26.7	-25.9	-20.3	-19.5	-19.8	-15.7	-20.5	-0.70
5000	-24.2	-19.6	-24.8	-24.0	-19.1	-18.3	-18.6	-14.5	-19.1	-0.60
4800	-22.8	-18.2	-23.6	-22.8	-18.1	-17.3	-17.5	-13.4	-17.9	-0.55
4600	-21.6	-17.0	-22.4	-21.6	-17.1	-16.3	-16.3	-12.2	-16.8	-0.75
4400	-19.8	-15.2	-21.0	-20.2	-15.7	-14.9	-15.0	-10.9	-15.8	-0.65
4200	-18.6	-14.0	-20.0	-19.2	-14.7	-13.9	-14.0	-9.9	-14.0	-0.45
4000	-17.5	-12.9	-18.8	-18.0	-13.7	-12.9	-12.6	-8.5	-13.1	-0.50
3800	-16.4	-11.8	-18.0	-17.2	-12.8	-12.0	-11.6	-7.5	-12.1	-0.45
3600	-15.4	-10.8	-17.2	-16.4	-11.9	-11.1	-10.6	-6.5	-11.2	-0.50
3400	-14.3	-9.7	-16.4	-15.6	-10.9	-10.1	-9.5	-5.4	-10.2	-0.55
3200	-13.2	-8.6	-15.1	-14.3	-9.8	-9.0	-8.6	-4.5	-9.1	-0.55
3000	-12.2	-7.6	-14.0	-13.2	-8.7	-7.9	-7.4	-3.3	-8.0	-0.40
2800	-11.2	-6.6	-13.1	-12.3	-8.4	-7.6	-6.9	-2.8	-7.2	-0.40
2600	-10.3	-5.7	-12.3	-11.5	-7.8	-7.0	-6.3	-2.2	-6.6	-0.40
2400	-9.4	-4.8	-11.4	-10.6	-7.2	-6.4	-5.4	-1.3	-5.8	-0.45
2200	-8.1	-3.5	-10.7	-9.9	-6.7	-5.9	-4.4	-0.3	-4.9	-0.40
2000	-7.1	-2.5	-9.8	-9.0	-6.2	-5.4	-3.6	0.5	-4.1	-0.35
1800	-6.5	-1.9	-8.8	-8.0	-5.5	-4.7	-2.9	1.2	-3.4	-0.35
1600	-6.0	-1.4	-7.9	-7.1	-4.9	-4.1	-2.3	1.8	-2.7	-0.35
1400	-5.3	-0.7	-6.8	-6.0	-4.3	-3.5	-1.9	2.2	-2.0	-0.30
1200	-4.6	0.0	-5.6	-4.8	-3.9	-3.1	-1.6	2.5	-1.4	-0.30
1000	-4.0	0.6	-4.6	-3.8	-3.4	-2.6	-1.3	2.8	-0.8	-0.40
800	-3.2	1.4	-3.4	-2.6	-2.8	-2.0	-0.9	3.2	0.0	-0.35
600	-2.4	2.2	-2.2	-1.4	-2.3	-1.5	-0.6	3.5	0.7	-0.35
400	-1.5	3.1	-1.2	-0.4	-1.7	-0.9	-0.4	3.7	1.4	-0.30
200	-0.7	3.9	-0.5	0.3	-0.8	0.0	-0.2	3.9	2.0	-0.30
0	0.0	4.6	0.0	0.8	0.0	0.8	0.0	4.1	2.6	
Mean gradient up to 4000 meters.....										-0.40

LOW AREAS IN WINTER.

Height in meters.	North.		East.		South.		West.		Mean.	
	ΔT	T	ΔT	T	ΔT	T	ΔT	T	T	ΔT 100
	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$
6000	-33.4	-26.5	-33.8	-29.6	-34.4	-27.8	-35.5	-29.0	-28.2	-0.75
5800	-31.7	-24.8	-32.4	-28.2	-33.2	-26.6	-34.0	-27.5	-26.7	-0.70
5600	-29.8	-22.9	-31.0	-26.8	-32.0	-25.4	-32.8	-26.3	-25.3	-0.65
5400	-28.2	-21.3	-29.7	-25.5	-30.8	-24.2	-31.8	-25.3	-24.0	-0.65
5200	-27.0	-20.1	-28.0	-23.8	-29.2	-22.6	-31.0	-24.5	-22.7	-0.50
5000	-26.1	-19.2	-26.6	-22.4	-28.3	-21.7	-30.3	-23.8	-21.7	-0.50
4800	-24.8	-17.9	-25.7	-21.5	-26.9	-20.3	-29.6	-23.1	-20.7	-0.55
4600	-23.8	-16.9	-24.3	-20.1	-25.8	-19.2	-28.8	-22.3	-19.6	-0.65
4400	-22.5	-15.6	-23.0	-18.8	-24.2	-17.6	-28.0	-21.5	-18.3	-0.65
4200	-21.1	-14.2	-21.4	-17.2	-22.8	-16.2	-26.9	-20.4	-17.0	-0.55
4000	-19.9	-13.0	-20.2	-16.0	-22.0	-15.4	-26.0	-19.5	-15.9	-0.55
3800	-18.7	-11.8	-19.0	-14.8	-21.0	-14.4	-24.7	-18.2	-14.8	-0.60
3600	-17.4	-10.5	-17.9	-13.7	-20.0	-13.4	-23.4	-16.9	-13.6	-0.65
3400	-16.0	-9.1	-16.5	-12.3	-18.8	-12.2	-22.2	-15.7	-12.3	-0.60
3200	-14.7	-7.8	-15.2	-11.0	-17.7	-11.1	-21.0	-14.5	-11.1	-0.55
3000	-13.5	-6.6	-14.2	-10.0	-16.6	-10.0	-19.7	-13.2	-10.0	-0.65
2800	-12.5	-5.6	-13.4	-9.2	-15.4	-8.8	-17.8	-11.3	-8.7	-0.65
2600	-11.8	-4.4	-12.6	-8.4	-14.2	-7.6	-15.9	-9.4	-7.4	-0.55
2400	-10.5	-3.6	-11.3	-7.1	-13.0	-6.4	-14.6	-8.1	-6.3	-0.60
2200	-9.8	-2.9	-10.2	-6.0	-11.8	-5.2	-13.1	-6.6	-5.1	-0.55
2000	-8.4	-1.5	-9.2	-5.0	-10.8	-4.2	-12.0	-5.5	-4.0	-0.45
1800	-7.8	-0.9	-8.0	-3.8	-9.8	-3.2	-11.0	-4.5	-3.1	-0.55
1600	-6.6	0.3	-7.0	-2.8	-8.8	-2.2	-10.0	-3.5	-2.0	-0.50
1400	-5.8	1.1	-5.9	-1.7	-7.7	-1.1	-8.8	-2.3	-1.0	-0.60
1200	-5.4	1.5	-4.7	-0.5	-6.4	0.2	-7.3	-0.8	0.2	-0.55
1000	-4.8	2.1	-3.2	1.0	-5.2	1.4	-6.0	0.5	1.3	-0.50
800	-3.6	3.3	-2.6	1.6	-4.2	2.4	-4.7	1.8	2.3	-0.50
600	-2.4	4.5	-1.8	2.4	-3.1	3.5	-3.8	2.7	3.3	-0.55
400	-1.3	5.6	-1.1	3.1	-2.1	4.5	-2.3	4.2	4.4	-0.50
200	-0.6	6.3	-0.5	3.7	-0.7	5.9	-0.9	5.6	5.4	-0.35
0	0.0	6.9	0.0	4.2	0.0	6.6	0.0	6.5	6.1	
Mean gradient up to 4000 meters.....										-0.55

tures of the sectors on the several levels, and the mean gradients per 100 meters. The gradients in the several sectors can be readily computed from the temperatures recorded for every 200-meter level up to 4000 meters. It should be noted that the data are entirely lacking for the north sector, except for the winter high areas, where three ascensions were available, and that observations in the central area are found only for the summer half of the year. This failure to record the temperature in the north sector is due to two causes, (1) the general passage of the centers of pressure to the north of Blue Hill, and (2), especially, the class of winds prevailing in that sector, which are usually unfavorable for kite ascensions.

For the European cyclones and anticyclones I have employed the data found in Grenander's paper. The temperature-falls in winter low and high areas at Hald and Berlin (Table 3) were taken from the tables and diagram without change. For these the results are recorded in Tables 3 and 4, where ΔT is the temperature-fall from the surface and T the corresponding temperature at every 200-meter level. In order to eliminate the diurnal variation from the surface mean temperatures, upon which every thing depends, it was necessary to interpolate a few of the missing night temperatures as in Table 5.

There is, of course, some uncertainty in this connection, but not nearly enough to modify the conclusions based upon these surface means. It is much better to obtain approximate mean temperatures for the 24 hours than to employ those belonging exclusively to the day hours. In the summer months the Hald and the Berlin temperatures were treated separately (see Table 4) and the adopted temperatures are the mean values of the two stations in the several 200-meter levels. The column T is the mean of the Hald and Berlin columns, with interpolations, and ΔT is computed from T .

TEMPERATURE-FALLS FROM THE SURFACE.

There are two ways in which the data of these tables can be conveniently arranged for discussion, (1) by exhibiting the temperature-falls on a diagram, and (2) by constructing the temperatures prevailing in the horizontal sections at different elevations. Fig. 1 represents the American and European temperature-falls in high and low areas, in winter and summer. It shows at a glance that the high areas (full lines) are not all cold as compared with the low areas, nor the low areas (dotted lines) all warm as compared with the high areas. On the other hand the north and west of the high areas are warm, while the south and east are cold, both winter and summer. In the lower levels, 1000 to 2000 meters, there is some entanglement of the gradient lines, and while there may be a tendency for the dotted lines of the low areas to cross the full lines of the high areas from right to left, there is nothing very decisive about the relation. In the American areas the temperatures of the high areas generally have a small gradient in the levels 1000 to 2000 meters, and there is, also, some indication of the same tendency in the European high areas. Attention is directed to chart 14, International Cloud Report, 1898, where a corresponding vertical change in the velocities was recorded, indicating that this stratum has a steady velocity throughout its depth in accordance with this local temperature distribution. In the winter both the European and American temperatures have about the same extreme differences, approximately $10^{\circ}C.$ above the 2000-meter level, though below it the American exceed the European by about $4^{\circ}C.$ The American circulation near the surface in winter is usually more active than the European. In the summer the American temperature divergence seems to be rather less, about 2° , than the European, though this difference may really be due to an inadequacy in the results of the observations in hand, and may be changed by increasing their number. On the whole there are indications that the temperature spread is wider at the 3000-meter level than at any height above or below, and this implies that the local circulation is greater at this level

corded in column four, and the temperature for each stratum is computed by adding the temperature-falls ΔT , as reduced to centigrade degrees, to the surface temperature T . At the head of each section group is given the number of ascensions employed, as 3 for north, 9 for east, 10 for south, etc. In the last columns of each general group are given the mean tempera-

FIG. 1.—American and European Temperature-falls, ΔT , in High and Low Areas, in Winter and Summer.

FIG. 3.—The Distribution of Temperature in the High and Low Pressure Areas of North America.

FIG. 4.—The Distribution of Temperature in the High and Low Pressure Areas of Europe.

TABLE 4.—*Temperature-falls at Hald and Berlin.*
HIGH AREAS IN SUMMER (APRIL TO SEPTEMBER).

Height in meters.	North.				East.				South.				West.				Mean.	
	Hald	Berlin	ΔT	T	Hald	Berlin	ΔT	T	Hald	Berlin	ΔT	T	Hald	Berlin	ΔT	T	T	ΔT 100
	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$
6000	-15.9	-17.5	-30.4	-16.7	-25.4	-23.1	-35.6	-24.3	-18.2	-19.5	-31.7	-18.9	-16.4	-16.9	-30.7	-16.7	-19.2	-0.65
5800			-29.2	-15.5			-34.0	-22.7			-30.5	-17.7			-29.4	-15.4	-17.9	-0.65
5600			-27.9	-14.2			-32.4	-21.1			-29.3	-16.5			-28.0	-14.0	-16.6	-0.65
5400			-26.7	-13.0			-31.0	-19.7			-28.1	-15.3			-26.7	-12.7	-15.3	-0.65
5200			-25.5	-11.8			-29.7	-18.4			-27.0	-14.2			-25.4	-11.4	-14.0	-0.60
5000	-9.7	-11.5	-24.3	-10.6	-18.5	-15.9	-28.5	-17.2	-12.2	-14.0	-25.9	-13.1	-9.6	-10.7	-24.2	-10.2	-12.8	-0.60
4800			-23.2	-9.5			-27.3	-16.0			-24.9	-12.1			-22.9	-8.9	-11.6	-0.60
4600			-22.1	-8.4			-26.1	-14.8			-23.8	-11.0			-21.6	-7.6	-10.4	-0.60
4400			-20.9	-7.2			-24.9	-13.6			-22.7	-9.9			-20.4	-6.4	-9.2	-0.60
4200			-19.7	-6.0			-23.7	-12.4			-21.6	-8.8			-19.3	-5.3	-8.1	-0.55
4000	-3.6	-6.0	-18.5	-4.8	-12.8	-9.6	-22.5	-11.2	-6.4	-9.0	-20.5	-7.7	-4.5	-3.9	-18.2	-4.2	-7.0	-0.50
3800			-17.3	-3.6			-21.3	-10.0			-19.2	-6.4			-17.2	-3.2	-6.0	-0.55
3600			-16.2	-2.5			-20.2	-8.9			-17.9	-5.1			-16.2	-2.2	-4.9	-0.50
3400			-15.3	-1.6			-19.3	-8.0			-16.7	-3.9			-15.2	-1.2	-3.9	-0.50
3200			-14.6	-0.9			-18.5	-7.2			-15.5	-2.7			-14.2	-0.2	-2.9	-0.45
3000	1.2	-2.0	-14.1	-0.4	-8.5	-4.5	-17.8	-6.5	-0.7	-2.6	-14.5	-1.7	0.1	1.6	-13.2	0.8	-2.0	-0.40
2800			-13.5	0.2			-17.3	-6.0			-13.6	-0.8			-12.1	1.9	-1.2	-0.40
2600			-12.9	0.8			-16.9	-5.6			-12.8	0.0			-11.0	3.0	-0.4	-0.40
2400			-12.3	1.4			-16.2	-4.9			-12.1	0.7			-10.0	4.0	0.4	-0.40
2200			-11.7	2.0			-15.3	-4.0			-11.2	1.6			-9.0	5.0	1.2	-0.45
2000	4.5	0.8	-11.0	2.7	-5.4	-0.3	-14.1	-2.9	3.7	1.5	-10.2	2.6	5.2	6.6	-8.1	5.9	2.1	-0.45
1800			-10.3	3.4			-12.9	-1.6			-9.3	3.5			-7.4	6.6	3.0	-0.50
1600			-9.6	4.1			-11.6	-0.3			-8.3	4.5			-6.6	7.4	4.0	-0.45
1400			-8.8	4.9			-10.2	1.1			-7.4	5.4			-5.8	8.2	4.9	-0.50
1200			-7.9	5.8			-8.9	2.4			-6.4	6.4			-5.0	9.0	5.9	-0.55
1000	7.4	6.1	-6.9	6.8	2.2	5.3	-7.6	3.8	7.2	7.5	-5.4	7.4	8.4	11.1	-4.2	9.8	7.0	-0.60
800			-5.7	8.0			-6.1	6.2			-4.4	8.4			-3.3	10.7	8.2	-0.55
600			-4.5	9.2			-4.6	6.7			-3.3	9.5			-2.5	11.5	9.3	-0.60
400			-3.2	10.5			-3.1	8.2			-2.2	10.6			-1.6	12.5	10.5	-0.60
200			-1.7	12.0			-1.6	9.7			-1.1	11.7			-0.8	13.2	11.7	-0.65
0	13.5	13.9	-0.0	13.7	9.1	13.5	0.0	11.3	10.5	15.1	0.0	12.8	11.4	16.5	0.0	14.0	13.0	
Mean gradient up to 4000 meters																		-0.50

LOW AREAS IN SUMMER.

6000	-17.0	-9.5	-30.1	-13.3	-18.9	-17.5	-35.5	-18.2	-22.0	-21.8	-35.4	-21.9	-24.5	-21.4	-36.0	-23.0	-18.1	-0.50
5800			-29.0	-12.2			-34.1	-16.8			-34.3	-20.8			-35.3	-21.9	-17.1	-0.45
5600			-27.9	-11.1			-32.8	-15.5			-33.1	-19.6			-34.1	-20.7	-16.2	-0.50
5400			-26.7	-9.9			-31.4	-14.1			-31.9	-18.4			-32.9	-19.5	-15.2	-0.50
5200			-25.6	-8.8			-30.1	-12.8			-30.7	-17.2			-31.7	-18.3	-14.2	-0.60
5000	-12.1	-3.5	-24.6	-7.8	-11.9	-11.3	-28.9	-11.6	-16.3	-15.9	-29.6	-16.1	-19.3	-14.8	-30.5	-17.1	-13.2	-0.55
4800			-23.8	-7.0			-27.7	-10.4			-28.5	-15.0			-29.2	-15.8	-12.1	-0.55
4600			-22.9	-6.1			-26.5	-9.2			-27.4	-13.9			-27.9	-14.5	-11.0	-0.60
4400			-21.8	-5.0			-25.4	-8.1			-26.3	-12.8			-26.5	-13.1	-9.8	-0.60
4200			-20.7	-3.9			-24.3	-7.0			-25.2	-11.7			-25.2	-11.8	-8.6	-0.55
4000	-7.5	2.0	-19.6	-2.8	-6.4	-5.4	-23.2	-5.9	-11.0	-10.2	-24.1	-10.6	-12.7	-8.2	-23.9	-10.5	-7.5	-0.50
3800			-18.7	-1.9			-22.2	-4.9			-23.1	-9.6			-22.8	-9.4	-6.5	-0.50
3600			-17.8	-1.0			-21.1	-3.8			-22.2	-8.7			-21.7	-8.3	-5.5	-0.55
3400			-16.8	0.0			-20.1	-2.8			-21.3	-7.8			-20.5	-7.1	-4.4	-0.50
3200			-15.8	1.0			-19.1	-1.8			-20.4	-6.9			-19.3	-5.9	-3.4	-0.50
3000	-3.1	7.0	-14.8	2.0	-1.5	0.0	-18.1	-0.8	-5.8	-5.9	-19.4	-5.9	-6.4	-3.0	-18.1	-4.7	-2.4	-0.50
2800			-14.0	2.8			-17.2	0.1			-18.3	-4.8			-16.9	-3.5	-1.4	-0.45
2600			-13.2	3.6			-16.3	1.0			-17.2	-3.7			-15.8	-2.4	-0.5	-0.50
2400			-12.3	4.6			-15.4	1.9			-16.1	-2.6			-14.7	-1.3	0.5	-0.55
2200			-11.4	5.4			-14.3	3.0			-15.0	-1.5			-13.6	-0.2	1.6	-0.60
2000	1.2	11.8	-10.4	6.4	4.0	4.4	-13.1	4.2	-0.2	-0.5	-13.9	-0.4	-0.9	2.7	-12.5	0.9	2.8	-0.60
1800			-9.2	7.6			-11.5	5.8			-12.8	0.7			-11.4	2.0	4.0	-0.60
1600			-8.0	8.8			-9.8	7.5			-11.6	1.9			-10.4	3.0	5.3	-0.65
1400			-6.7	10.1			-8.1	9.2			-10.4	3.1			-9.2	4.2	6.6	-0.65
1200			-5.4	11.4			-6.6	10.7			-9.1	4.4			-8.0	5.4	7.9	-0.65
1000	6.3	18.7	-4.3	12.5	10.2	13.7	-5.3	12.0	5.2	6.2	-7.8	5.7	4.3	9.0	-6.7	6.7	9.2	-0.65
800			-3.4	13.4			-4.2	13.1			-6.4	7.1			-5.3	8.1	10.4	-0.60
600			-2.5	14.3			-3.2	14.1			-4.8	8.7			-4.0	9.4	11.6	-0.60
400			-1.7	15.1			-2.1	15.2			-3.3	10.2			-2.7	10.7	12.9	-0.65
200			-0.8	16.0			-1.0	16.3			-1.7	11.8			-1.4	12.0	14.1	-0.60
0	12.4	21.2	0.0	16.8	14.9	19.6	0.0	17.3	13.0	14.0	0.0	13.5	12.4	14.4	0.0	13.4	15.3	
Mean gradient up to 4000 meters																		-0.57

than above or below it, a result indicated by the observed velocities as shown on chart 68 of my Cloud Report. It is hardly possible to consider the temperatures for the upper levels, 4000 to 6000 meters, as sufficiently reliable to base special emphasis upon the wide divergence of the European temperatures in those levels, and the lines should probably be drawing together more rapidly than here shown. From other considerations it may be inferred that the temperature differences generally disappear in the neighborhood of the cirrus levels 9000 to 10,000 meters. While there is a tendency in the lower levels for the temperatures of the high areas to diminish less rapidly than those of the low areas, there is yet but little to

justify the view that anything like an inversion of temperature occurs, such as Professor Hann found in mountain stations and ascribed to dynamic actions, or such as Bjerknes and Clayton assume to exist in their theories of cold-center and warm-center cyclones superposed upon each other. This will be seen more clearly in the exhibit of figs. 3 and 4. We may safely infer from fig. 1 that there is little to distinguish the temperature distribution in European cyclones and anti-cyclones from that prevailing in the American high and low areas.

Using the same temperature-falls we now discuss the relation according to actual warm and cold areas, as shown in

fig. 2, where the dotted lines indicate the warm areas and the full lines the cold areas. Hence we adopt the following arrangement: (See Table 6.)

TABLE 5.—Surface temperatures in centigrade degrees for day and night at Hald and Berlin.

HIGH AREAS.									
Station.		Winter.				Summer.			
		N.	E.	S.	W.	N.	E.	S.	W.
Hald	{ day	5.1	8.8	1.1	1.8	14.9	10.4	14.0	12.6
	{ night	2.1	(0.0)	—1.7	5.1	(12.0)	(7.8)	(6.9)	(10.2)
Berlin	{ day	6.6	1.7	1.9	4.2	17.1	16.6	18.5	19.0
	{ night	4.6	—2.5	1.8	5.9	10.6	10.3	11.6	14.0
Means		4.6	0.8	0.8	4.1	13.7	11.3	12.8	14.0
LOW AREAS.									
		N.	E.	S.	W.	N.	E.	S.	W.
Hald	{ day	6.1	5.5	6.2	4.6	14.6	15.8	14.0	12.8
	{ night	(4.0)	3.7	6.0	5.8	10.1	14.0	12.0	(12.0)
Berlin	{ day	12.8	2.7	8.6	9.2	26.5	20.9	15.5	14.9
	{ night	4.7	4.7	(6.6)	(6.6)	15.8	18.3	12.6	(14.0)
Means		6.9	4.2	6.6	6.5	16.8	17.3	13.5	13.4

TABLE 6.—Distribution of the sectors by warm and cold areas.

Blue Hill, winter.		Hald and Berlin, winter.	
Warm areas.	Cold areas.	Warm areas.	Cold areas.
High north. High west. Low east. Low south.	High east. High south. Low west. Low north.	High north. High south. High west. Low north.	High east. Low east. Low south. Low west.
Blue Hill, summer.		Hald and Berlin, summer.	
High north. High west. Low east. Low south.	High east. High south. Low north. Low west.	High north. High west. Low north. Low east.	High east. High south. Low south. Low west.

This summary shows that as a whole the warm and cold temperatures are about equally distributed between the high areas and low areas, especially in the levels above 1000 meters. Below that level, in the stratum near the ground, there is much less regularity in the distribution, and this probably means that the true cyclonic and anticyclonic action is disturbed by coming in contact with the ground, and by the adjustment to surface conditions. *It also shows that the surface temperatures are not entirely reliable as records of the real distribution prevailing in the free air, and it implies that for accurate forecasting it will be necessary to change from the exclusive use of surface temperature charts to those determined by observations in the free air at somewhat moderate elevations, such as can be easily obtained by kite flights up to 1000 or 2000 meters.*

TEMPERATURE DIFFERENCES AT THE SAME ELEVATION.

The results of the computations may be exhibited even more clearly by plotting the temperatures on the several 1000-meter levels, and drawing a line to separate the high temperature from the low temperature regions. Fig. 3 shows the distribution for the American and fig. 4 for the European cyclonic and anticyclonic regions. The loss of the data in the north sector of the American cyclone makes this line of demarcation uncertain in that area, but by analogy with the European data and from what we know of conditions at the surface it is probably quite like that indicated in the diagram. In spite of the fact that we have been using an insufficient amount of observations to render the temperature distribution entirely normal, it is yet evident that the distribution is fundamentally the same in the American and European circulation. *There is an inflow of cold air from the northwest between the centers of baro-*

metric high and low pressure, and an inflow of warm air from the south, likewise between the centers of low and high pressure. There are no cold-center anticyclones in any level, nor any warm-center cyclones in any level. There is no inversion of type from the surface to the highest level reached, with the possible exception of the surface and the 1000-meter level of the European cyclones in the winter. I am at a loss to explain this last result, and suspect that it may be due to our imperfect elimination of the diurnal temperature variation from the data contained in Grenander's paper, or in the Hald and Berlin reports. This exception seems to constitute the basis for the claim that has been advanced for Professor Hann's theory that the warm-center cyclone below is replaced by a cold-center cyclone above, and that this inversion of temperature implies a dynamic system as the source of the level cooling. The fact is that the cool northwest winds in the upper levels follow the stream lines marked out in the diagrams of the Weather Bureau International Cloud Report, as given in chart 15, or in the MONTHLY WEATHER REVIEW, March, 1902. They become more and more sinuous in descending from the cirrus levels to the surface, or below the strato-cumulus level, 3000 meters, curling more decidedly in an anticyclonic rotation with a tendency to divide into two branches. The southerly winds in the same way divide into two branches one curling into the cyclone on the northeast and the other into the anticyclone on the northwest of the respective centers.

The mechanical cyclone and anticyclone, with centers of high and low pressure at the boundary between these cold and warm currents, are the dynamic effect of the vertical and horizontal motion of these currents of different temperature, just as has been explained in my previous papers. The observed vectors of velocity are such as to produce or accompany the temperature distribution here described, and the cold and the warm currents having different temperatures bear within themselves the source of the energy of storms. The fact is that storms are produced by horizontal convection more than by vertical convection. The latent heat of condensation is an additional source of energy, and the underflowing of warm currents beneath the eastward drift is another small source of energy, but these are distinctly subordinate to the horizontal or lateral convection between these counterflowing masses or sheets of air. The cyclone and anticyclone are the dynamic effects of this thermodynamic energy, and this function constitutes the true problem in meteorology for study. It is, therefore, evident that the different statements found in the literature of the subject regarding the cyclone being cooler than the anticyclone in the upper levels, have been imperfect and rough efforts to reach the facts here shown. Since the counterflow does have somewhat different configurations in the several levels, there is a difference of temperature to be obtained on proceeding from the surface upward over the same sector. But it is not proper to compare the temperatures of cyclones as a whole, from level to level, nor of anticyclones, without careful discrimination as to the subareas which are involved. It is important, likewise, to obtain the gradients over the warm and cold areas, separately, rather than over the cyclones and anticyclones taken as wholes, as was done in Tables 1, 2, 3, 4.

VERTICAL TEMPERATURE GRADIENTS PER 100 METERS.

Taking the mean gradients over the high areas and the low areas for the American and European temperatures, and limiting the means to the 4000-meter level for the sake of the comparison, we obtain from Tables 1, 2, 3, 4:

Winter.			Summer.		
	High areas.	Low areas.		High areas.	Low areas.
American	° F. — 0.42	° F. — 0.43	American	° F. — 0.62	° F. — 0.63
European	— 0.40	— 0.55	European	— 0.50	— 0.57

TABLE 7.—Temperatures and gradients in the American and European cyclones and anticyclones arranged by the warm and cold areas.

Height in meters.	Blue Hill, winter.				Hald and Berlin, winter.				Blue Hill, summer.				Hald and Berlin, summer.			
	Warm.		Cold.		Warm.		Cold.		Warm.		Cold.		Warm.		Cold.	
	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.
6000					—24.9	—0.75	—29.0	—0.70					—16.2	—0.60	—22.0	—0.60
5800					—23.4	—0.80	—28.5	—0.70					—15.0	—0.65	—20.8	—0.65
5600					—21.8	—0.70	—27.1	—0.70					—13.7	—0.65	—19.5	—0.65
5400					—20.4	—0.65	—25.7	—0.65					—12.4	—0.60	—18.2	—0.60
5200					—19.1	—0.60	—24.2	—0.60					—11.2	—0.55	—17.0	—0.55
5000					—17.9	—0.55	—23.0	—0.55					—10.1	—0.55	—15.9	—0.60
4800					—16.8	—0.60	—21.9	—0.60					—9.0	—0.60	—14.7	—0.55
4600					—15.6	—0.65	—20.7	—0.60					—7.8	—0.55	—13.6	—0.60
4400					—14.3	—0.65	—19.5	—0.60					—6.7	—0.55	—12.4	—0.60
4200					—13.0	—0.60	—18.3	—0.55					—5.6	—0.60	—11.2	—0.60
4000	—11.9	—0.50	—17.6	—0.55	—11.8	—0.50	—17.2	—0.50	—4.3	—0.70	—9.2	—0.70	—4.4	—0.50	—10.0	—0.55
3800	—10.9	—0.45	—16.5	—0.55	—10.8	—0.55	—16.2	—0.55	—2.9	—0.60	—7.8	—0.65	—3.4	—0.50	—8.9	—0.55
3600	—10.0	—0.50	—15.4	—0.50	—9.7	—0.55	—15.1	—0.55	—1.7	—0.65	—6.5	—0.75	—2.4	—0.50	—7.8	—0.55
3400	—9.0	—0.50	—14.4	—0.60	—8.6	—0.55	—14.0	—0.60	—0.4	—0.60	—5.0	—0.70	—1.4	—0.50	—6.7	—0.50
3200	—8.0	—0.55	—13.4	—0.55	—7.5	—0.55	—12.8	—0.60	0.8	—0.65	—3.6	—0.70	—0.4	—0.40	—5.7	—0.50
3000	—6.9	—0.45	—12.3	—0.50	—6.4	—0.40	—11.6	—0.60	2.1	—0.55	—2.2	—0.65	0.4	—0.45	—4.7	—0.45
2800	—6.0	—0.45	—11.3	—0.45	—5.6	—0.40	—10.4	—0.60	3.2	—0.60	—0.9	—0.70	1.3	—0.40	—3.8	—0.45
2600	—5.1	—0.45	—10.4	—0.50	—4.8	—0.40	—9.2	—0.55	4.4	—0.55	0.5	—0.65	2.1	—0.40	—2.9	—0.45
2400	—4.2	—0.50	—9.4	—0.40	—4.0	—0.40	—8.1	—0.60	5.5	—0.60	1.8	—0.65	3.0	—0.45	—2.0	—0.50
2200	—3.2	—0.45	—8.6	—0.45	—3.2	—0.45	—6.9	—0.50	6.7	—0.55	3.1	—0.65	3.9	—0.45	—1.0	—0.55
2000	—2.3	—0.50	—7.7	—0.40	—2.3	—0.35	—5.9	—0.50	7.8	—0.60	4.4	—0.60	4.8	—0.55	0.1	—0.55
1800	—1.3	—0.50	—6.9	—0.40	—1.6	—0.35	—4.9	—0.50	9.0	—0.50	5.6	—0.70	5.9	—0.55	1.2	—0.55
1600	—0.3	—0.25	—6.1	—0.40	—0.9	—0.35	—3.9	—0.60	10.0	—0.55	7.0	—0.65	7.0	—0.55	2.3	—0.60
1400	0.2	—0.15	—5.4	—0.35	—0.2	—0.20	—2.7	—0.60	11.1	—0.50	8.3	—0.65	8.1	—0.55	3.5	—0.60
1200	0.5	—0.20	—4.8	—0.30	0.2	—0.25	—1.5	—0.60	12.1	—0.45	9.6	—0.40	9.2	—0.55	4.7	—0.60
1000	0.9	—0.20	—4.6	—0.10	0.7	—0.40	—0.3	—0.55	13.0	—0.40	10.4	—0.40	10.3	—0.50	5.9	—0.65
800	1.3	—0.35	—4.3	—0.15	1.5	—0.35	0.8	—0.50	13.8	—0.55	11.2	—0.50	11.3	—0.50	7.2	—0.70
600	2.0	—0.50	—3.8	—0.25	2.2	—0.35	1.8	—0.55	14.9	—0.55	12.2	—0.60	12.3	—0.50	8.6	—0.65
400	3.0	—0.65	—2.9	—0.35	2.9	—0.30	2.9	—0.40	16.0	—0.70	13.4	—0.60	13.3	—0.55	9.9	—0.70
200	4.2	—0.65	—1.8	—0.65	3.5	—0.30	3.7	—0.35	17.4	—0.65	14.6	—0.55	14.4	—0.55	11.3	—0.75
0	5.5		—0.5		4.1		4.4		18.7		15.7		15.5		12.8	

The American mean gradient is about the same for high and low areas at all seasons of the year, though the summer gradient is greater than the winter in the ratio of 3 to 2. This is a function of the observed difference of velocity for these two seasons. The European gradient is larger in cyclones than in anticyclones for both seasons, and there is not so much difference between them in the summer as in the winter. These variations depend upon the local conditions which control the origin and mixing of the countercurrents themselves. Surveying the gradient lines as a whole, I believe that they show that, after escaping from the surface confusion, the lines spread gradually up to 3000 meters, or the strato-cumulus level, where the difference is a maximum and that they then draw slowly together up to about the cirrus level where the eastward drift is usually undisturbed. This conforms to my solution obtained for the local cyclonic velocities, which showed that the maximum gyration is in the cirro-cumulus level, and that it diminishes downward and upward without reversal.⁵ The discussion of these velocities and temperatures, together with the corresponding pressures will be undertaken in a later paper of this series. It is now evident that any adiabatic solution of the problem is inadequate, and that the omission of the heat added, $\int Q=0$, is fatal to a satisfactory discussion of natural cyclones and anticyclones, because the observed gradients differ from the adiabatic gradient, 0.987 degree centigrade per 100 meters. Furthermore, the surrounding of fixed masses of air with artificial boundaries for the sake of a ready integration, also takes the problem out of the class of those required in meteorology, and makes merely special ideal cases which are out of our consideration. It has seemed to me proper to wait for the acquisition of approximate values of the velocity, temperature, and pressure before trying to do anything with the analytical solutions. Since it is the divergence of the local temperatures away from the mean temperature of the atmosphere which produces the local circulations, we should evidently compute the gradients in the warm and cold areas regardless of their distribution around the centers of pressure.

⁵ Compare chart 68, International Cloud Report, figs. 6, 7; and Tables 11, 12, Monthly Weather Review, March, 1902.

TABLE 8.—Temperature differences, cold minus warm areas.

Height.	Winter.		Summer.	
	Blue Hill.	Hald and Berlin.	Blue Hill.	Hald and Berlin.
	° F.	° F.	° F.	° F.
6000		—5.0		—5.8
5000		—5.1		—5.8
4000	—5.7	—5.4	—4.9	—5.6
3000	—5.4	—5.2	—4.3	—5.1
2000	—5.4	—3.6	—3.4	—4.7
1000	—5.5	—1.0	—2.6	—4.4
0	—6.0	+0.3	—3.0	—2.7

VERTICAL TEMPERATURE GRADIENTS IN THE WARM AND COLD AREAS.

By taking the mean temperatures at the different levels, arranged according to the warm and cold area groups indicated in Table 6, we obtain the temperatures and gradients of Tables 7 and 8. There are several results which can be readily seen on the face of the tables: (1) The temperatures at Blue Hill, Hald and Berlin are in sufficient agreement as to the absolute values in the several levels, to enable us to infer that the American and European systems are in harmony so far as the temperature is concerned. (2) The difference between the temperatures in the warm and cold areas is about 5.4° on the 4000-meter level, and it diminishes toward the surface, except at Blue Hill in winter. For Hald and Berlin in winter there is the anomalous inversion already mentioned. For summer, at all stations, the temperature difference is about 3° in the lower levels. (3) The gradient at Blue Hill in winter is —0.65° near surface, —0.20° at the 1000-meter level, and —0.50° at 4000 meters. Hald and Berlin show a comparatively steady increase of the gradient from the surface—0.35°, to —0.75° at 6000 meters, with a slight diminution in the 1000 to 2000-meter stratum for the warm areas; in the cold areas the gradient is about —0.60° in the upper levels throughout a deep stratum. In summer the gradient at the surface is about —0.60°, at the 1000-meter level —0.40°, and at 4000 meters —0.65, at Blue Hill; at Hald and Berlin there is a diminution of the gradient up to the 3000-meter level and an increase up to the 6000-meter level for warm and cold areas. These facts are of special significance in the thermodynamics of

cyclonic circulations. These *temperature* gradients may profitably be compared with the gradients in high or low *pressure* areas taken as a whole. The departure of the temperature in each sector from the prevailing mean air temperature is evidently the basis for a thermodynamic discussion of the cyclonic problem.

RESULTS.

We conclude that there is no fundamental difference in the structure of American and European cyclones and anticyclones. The observed temperature distribution is in harmony with the observed atmospheric currents, and is due to an intermixture of currents from warm and cold latitudes, the energy of storms being thus referred to the heat transported from different latitudes. The pressure centers of motion occur on the boundaries of these countercurrents, and thus represent the dynamic effects of the thermodynamic energy. Instead of vertical convection being the primary cause of storms it is rather horizontal convection, interchange of heat energy on the same levels, as suggested in my preceding papers. There is no evidence of the superposition of cold-center cyclones upon warm-center cyclones, as expounded by Clayton or by Bjerknes and Arrhenius, nor are there purely dynamic vortices in a rapid stream as supposed by Hann, nor are there cyclonic vortices caused by atmospheric islands of high pressure obstructing a rapidly flowing eastward drift as explained by Shaw, or by Hildebrandsson in his report to the International Committee, 1905.

ATMOSPHERIC ELECTRICITY.

By GEORGE C. SIMPSON, M. S., Lecturer in meteorology in the University of Manchester, England. Dated January 17, 1906.

The study of meteorology may be pursued with two entirely different ends in view. We may pursue it for utilitarian purposes, or we may pursue it as a pure science for the knowledge to be derived from it. It must be admitted that there are a number of so-called meteorologists whose point of view is the former, and whose highest ambition is the production of an almanac giving the state of the weather for each day a year in advance. Although some of the great advances of knowledge which have been of practical use to mankind have been made by the utilitarian student, yet experience has shown that without the pure scientist little real advance can be made. The true men of research, working for the sake of science itself, have always been the pioneers to open up new country and reveal new treasures, which the utilitarian has then appropriated and used. In meteorology we must not expect and seek for merely practical knowledge, but must investigate the atmosphere in a truly scientific manner, considering no phenomenon found in it to be unworthy of investigation. Thus although the electrical conditions of the atmosphere on a fine day are insignificant in comparison with the great motions of the atmosphere, or with the local conditions which determine the climate and weather of any place, yet meteorology can not be complete without a true knowledge of these conditions. While meteorologists must always be more concerned with the great changes in the motion, temperature, pressure, and humidity of the air, they must also not forget that electricity plays an important part in natural phenomena. Since the time of Franklin great progress has been made in our knowledge of the electrical conditions of the atmosphere, but so far the progress has led to no utilitarian results, and the problems of the thunderstorm are not yet solved. In the present article an attempt is made to sketch, in as few words and as simply as possible, the lines along which research has traveled.

One of the first lessons we learn on being introduced to the science of electricity is that a positively charged body placed between two others, one having a positive and the other a negative charge, will tend to move toward the latter. This we are told is due to the "electrical field" set up by the oppositely charged bodies.

The experiments of Franklin showed that such an electrical field exists in the atmosphere during thunderstorms. Later observers have shown that not only during thunderstorms, but also under normal circumstances, there is an electrical field in the lower atmosphere, such that a positively charged body would be attracted toward the surface of the earth. The electrical field of the lower atmosphere points to the permanent presence of a layer of negative electricity on the surface of the earth with a corresponding positive charge somewhere above the surface, but exactly where this positive charge is has been the subject of much controversy.

For a long time it was thought that the negative charge on the earth is a charge left there when the earth first became a separate member of the solar system. If that had been so then the corresponding charge of positive electricity would be somewhere right outside the earth and its surrounding atmosphere, in fact somewhere in cosmical space. We should thus have expected an electrical field, similar to that found near the surface of the earth, extending with only slightly diminished intensity to heights much greater than those of our atmosphere; but observations made in balloons have proved most conclusively that the electrical field found near the surface has nearly disappeared at a height of four and one-half miles (reached by Gerdien, November 5, 1903). This means that the outside positive charge corresponding to the negative charge on the surface of the earth is not outside the atmosphere, but is distributed through the whole mass of air in the lower atmosphere.

This result leads us to consider under what conditions electricity can exist in the atmosphere. Until 1900 the conception was firmly held by physicists that the air of the atmosphere in its normal state is a perfect nonconductor of electricity, and that if a charged body, perfectly insulated, were placed in air quite free from dust the charge would be retained indefinitely. That such a body never does permanently keep its charge was always ascribed to the presence of dust in the air. It was supposed that particles of dust coming into contact with the body take a part of the charge and are then repelled, and that thus in course of time the charge is in this way entirely dissipated on to the dust of the air. But in 1900 Elster and Geitel in Germany and C. T. R. Wilson in England showed that not only does a perfectly insulated body in perfectly pure air lose its charge, but that the presence of dust diminishes the rate of loss instead of increasing it.

Just before this time new ideas had been formed as to what takes place during the discharge of electricity through gases. It had been demonstrated that most, if not all, the phenomena then known to accompany the discharge of electricity through gases could be explained by assuming that an electrically neutral molecule of a gas can be split up into two other molecules or two atoms, each of which carries an electrical charge, one negative and the other positive. To these charged molecules the name "ion" had been given, an "ion" being understood to be any small material particle, generally of molecular dimensions, which carries a charge of electricity.

Elster, Geitel, and Wilson at once pressed this theory into service to explain the results of their experiments. They assumed that a small proportion of the molecules of ordinary air are always being split up into ions. Thus when a charged body is introduced into air the electricity on it attracts ions of the opposite sign which neutralize the charge, or in other words the charge is dissipated. Numerous experiments were made to prove or disprove this theory, but it has withstood all the tests and is now generally accepted. Elster and Geitel proceeded to work out in full the bearings of this new theory on the problems of atmospheric electricity. It at once became obvious that if there are always both kinds of ions in the atmosphere the negative charge on the earth will attract the positive ions toward itself, become neutralized and so disap-

STUDIES ON THE THERMODYNAMICS OF THE ATMOSPHERE.
II.—COORDINATION OF THE VELOCITY, TEMPERATURE, AND PRESSURE IN THE CYCLONES AND ANTICYCLONES OF EUROPE AND NORTH AMERICA.

By Prof. FRANK H. BIGELOW.

THE ADOPTED MEAN TEMPERATURES AND GRADIENTS ON THE 1000-METER LEVELS FOR AMERICAN AND EUROPEAN CYCLONES AND ANTICYCLONES.

In order that we may have a suitable example for discussion in the application of the thermodynamic formulæ to the atmosphere, it will be profitable to adopt an average system of temperature distributions in cyclones and anticyclones, derived from the observations that have been made available by the computations of the preceding paper of this series. The practical difficulty of securing, by numerous ascensions at the same time, a sufficiently large number of accurate simultaneous observations in the several subareas around the barometric centers, for all the levels required up to 10,000 meters, is very great when one special temperature disturbance is to be observed, as in an individual storm. It would be desirable to make numerous simultaneous ascensions from different points of the same cyclone, as has been done in Europe, if it is possible to do so from many stations, say for 25 to 40 in number, but the result will then be to give the conditions in only one case, and it would of course require many similar groups of such ascensions to enable us to secure what may be accepted as the average or normal type of cyclonic temperature distribution. Repeated ascensions at a single station, where the facilities for the work are generally much more adequately elaborated, accomplish the same purpose by allowing the kites to remain aloft, and thus permitting the cyclone to drift past the instruments which register the varying changes of temperature, pressure, humidity, and wind velocity. By utilizing the numerous ascensions of kites and balloons made at Hald, Berlin, and Blue Hill, we can secure the approximate normal or average state of the temperatures with which to explain and test the proposed thermodynamic system. Owing to the incompleteness of our available data, it is necessary to assume some extensions of the probable temperatures beyond the actual observations, but it can be done without bringing into doubt the main principles involved in this discussion. The further acquisition of observed data by work in the future will correct any numerical errors that may now arise from this procedure. In order to establish the theory of the energy of storms which I have proposed, it seems proper to use the best available data in these preliminary computations, rather than wait for more completely satisfactory compilations of temperature observations. It will, also, facilitate a correct interpretation of the results of future balloon and kite ascensions, if we can show that our theory is capable of suitably explaining the present adopted mean system of temperature distributions in cyclones and anticyclones. We can infer from the preceding paper that the American and European observations at the three given stations in the lower levels approach so closely together in their numerical details on the 4000-meter level, that we shall be justified in adopting the European observations above 4000 meters as a fair representation of the conditions in the North American atmosphere, which, however, remains to be practically explored in the higher levels.

In order to concentrate the work of computation which will follow, we extract from Tables 1, 2, 3, 4, the temperatures, *T*, in centigrade degrees, for the several 1000-meter levels, as given in the last column of each quadrant. These temperatures appear in the left-hand section of Tables 9 and 10, which give the temperatures and variations of temperature on each 1000-meter level in American and European cyclones and anticyclones for the winter and for the summer. It should be noted that we have been obliged to introduce certain values for the north quadrant in the American data, because direct

observations are entirely lacking, except in the winter high areas. The values here adopted have been the outcome of considerable labor, and they may be found to need some modification as the result of other direct observations. I have been guided in the determination of the interpolated temperatures by three different considerations, (1) the relations of the center areas to the north areas in general, (2) the comparison of the European data with the American, and finally, (3) the balance of the entire system, consisting of the four sectors of the high areas and the four sectors of the low areas taken in connection with the mean temperatures and gradients of the atmosphere without regard to any cyclonic disturbances. Thus, having adopted the mean temperatures and gradients of Table 11 and the mean variations in seven sectors of Table 9, it is easy to compute the variation in any missing quadrant, as the north. A complete balance must evidently be maintained, because the sum of the excess and defect of the temperatures in the several areas on a given level of the cyclone and anticyclone at any elevation, as 1000 meters, 2000 meters, etc., must be equal to zero. However, in these papers, as we are concerned only with approximate relative disturbances of the temperature, and since individual cyclonic disturbances differ widely from any normal type that may be adopted, such interpolations may be admitted for the purpose of discussion.

TABLE 9.—Temperatures and variations of temperature on each 1000-meter level in American cyclones and anticyclones.

AMERICAN WINTER HIGH AREAS.—From Table 1.										
Height in meters.	Temperatures <i>T</i> .				Means, <i>T</i> ₀	Variations $-\Delta T$.				Mean $-\Delta T$.
	N.	E.	S.	W.		N.	E.	S.	W.	
	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.
6000					-26.0					
5000					-19.5					
4000	-7.9	-18.8	-17.9	-7.6	-14.0	+6.1	-4.8	-3.9	+6.4	
3000	-3.9	-12.7	-12.6	-3.1	-9.0	+5.1	-3.7	-3.6	+5.9	
2000	-1.2	-7.1	-8.3	1.0	-4.8	+3.6	-2.3	-3.5	+5.8	
1000	1.8	-3.5	-3.6	4.5	-1.5	+3.3	-2.0	-2.1	+6.0	
0	3.7	2.0	-0.7	9.0	2.8	+0.9	-0.8	-3.5	+6.2	
AMERICAN WINTER LOW AREAS.—From Table 1.										
6000					-26.0					
5000					-19.5					
4000	(-12.0)	-13.2	-18.7	-16.0	-14.0	+2.0	+0.8	-4.7	-2.0	3.84
3000	(-7.0)	-8.1	-12.6	-11.6	-9.0	+2.0	+0.9	-3.6	-2.6	3.43
2000	(-2.0)	-3.0	-6.3	-7.8	-4.8	+2.8	+1.8	-1.5	-3.0	3.04
1000	(-2.0)	-2.7	-0.1	-6.8	-1.5	-0.5	-1.2	+1.4	-5.3	2.73
0	(2.0)	2.8	6.3	-2.7	2.8	-0.8	0.0	+3.5	-5.5	2.65
Mean										3.14
AMERICAN SUMMER HIGH AREAS.—From Table 2.										
6000					-20.6					
5000					-14.1					
4000	-3.0	-8.4	-9.6	-4.8	-7.1	+4.1	-1.3	-2.5	+2.3	
3000	3.6	0.1	-0.6	2.0	0.4	+3.2	-0.3	-1.0	+1.6	
2000	9.4	7.1	5.8	8.1	6.4	+3.0	+0.7	-0.6	+1.7	
1000	13.6	13.0	11.2	12.1	11.8	+1.8	+1.2	-0.6	+0.3	
0	17.9	16.7	14.5	18.1	17.2	+0.7	-0.5	-2.7	+0.9	
AMERICAN SUMMER LOW AREAS.—From Table 2.										
6000					-20.6					
5000					-14.1					
4000	-11.4	-6.0	-3.6	-9.0	-7.1	-4.3	+1.1	+3.5	-1.9	2.63
3000	-1.4	-0.3	3.1	-3.3	0.4	-1.8	-0.7	+2.7	-3.7	1.88
2000	4.1	4.8	8.9	3.0	6.4	-2.3	-1.6	+2.5	-3.4	1.98
1000	9.5	10.2	16.0	8.8	11.8	-2.3	-1.6	+4.2	-3.0	2.00
0	14.7	15.6	23.2	16.9	17.2	-2.5	-1.6	+6.0	-0.3	1.53
Mean										2.00

We next take the mean temperatures of all the high and low areas for the four available systems, the American winter and summer up to 4000 meters, and the European winter and summer up to about 10,000 meters, and set them in the middle column of Tables 9 and 10, as well as in Table 11, the adopted mean temperatures and gradients on the 1000-meter levels for American and European cyclones and anticyclones. It will be seen that at the 4000-meter level the temperatures are almost exactly equal in the two regions, for winter -14.0°

TABLE 10.—*Temperatures and variations of temperature on each 1000-meter level in European cyclones and anticyclones.*

EUROPEAN WINTER HIGH AREAS.—From Table 3.										
Height in meters.	Temperatures T .				Means, T_0 .	Variations $-\Delta T$.				Mean $-\Delta T$.
	N.	E.	S.	W.		N.	E.	S.	W.	
	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$
6000.....	-27.0	-33.2	-24.5	-21.6	-27.4	+0.4	-5.8	+2.9	+5.8	
5000.....	-19.4	-24.0	-18.3	-14.5	-20.4	+1.0	-3.6	+2.1	+5.9	
4000.....	-12.9	-18.0	-12.9	-8.5	-14.5	+1.6	-3.5	+1.6	+6.0	
3000.....	-7.6	-13.2	-7.9	-3.3	-9.0	+1.4	-4.2	+1.1	+5.7	
2000.....	-2.5	-9.0	-5.4	0.5	-4.1	+1.6	-4.9	+1.3	+4.6	
1000.....	0.6	-3.8	-2.6	2.8	0.3	+0.3	-4.1	-2.9	+2.5	
0.....	4.6	0.8	0.8	4.1	4.3	+0.3	-3.5	-3.5	-0.2	
EUROPEAN WINTER LOW AREAS.—From Table 3.										
6000.....	-26.5	-29.6	-27.8	-29.0	-27.4	+0.9	-2.2	-0.4	-1.6	2.50
5000.....	-19.2	-22.4	-21.7	-23.8	-20.4	+1.2	-2.0	-1.3	-3.4	2.56
4000.....	-13.0	-16.0	-15.4	-19.5	-14.5	+1.5	-1.5	-0.9	-5.0	2.70
3000.....	-6.6	-10.0	-10.0	-13.2	-9.0	+2.4	-1.0	-1.0	-4.2	2.63
2000.....	-1.5	-5.0	-4.2	-6.5	-4.1	+2.5	-0.9	-0.1	-1.4	2.18
1000.....	2.1	1.0	1.4	0.5	0.3	+1.8	+0.7	+1.1	+0.2	1.70
0.....	6.9	4.2	5.6	6.5	4.3	+2.6	-0.1	+2.3	+2.2	1.83
Mean from 0 to 4000 meters, 2.21										
EUROPEAN SUMMER HIGH AREAS.—From Table 4.										
6000.....	-15.7	-24.3	-18.9	-16.7	-19.1	+2.4	-5.2	+0.2	+2.4	
5000.....	-10.6	-17.2	-13.1	-10.2	-13.0	+2.4	-4.2	-0.1	+2.8	
4000.....	-4.8	-11.2	-7.7	-4.2	-7.2	+2.4	-4.0	-0.5	+3.0	
3000.....	-0.4	-6.5	-1.7	0.8	-2.1	+1.7	-4.4	+0.4	+2.9	
2000.....	2.7	-2.9	2.6	5.9	2.2	+0.5	-6.1	+0.4	+3.7	
1000.....	6.8	3.8	7.4	9.8	8.1	-1.3	-4.3	-0.7	+1.7	
0.....	13.7	11.3	12.8	14.0	14.1	-0.4	-2.8	-1.3	-0.1	
EUROPEAN SUMMER LOW AREAS.—From Table 4.										
6000.....	-13.3	-18.2	-21.9	-23.0	-19.1	+5.8	+0.9	-2.8	-3.9	2.95
5000.....	-7.8	-11.6	-16.1	-17.1	-13.0	+5.2	+1.4	-3.1	-4.1	2.91
4000.....	-2.8	-5.9	-10.6	-10.5	-7.2	+4.4	+1.3	-3.4	-3.3	2.79
3000.....	2.0	-0.8	-5.9	-4.7	-2.1	+4.1	+1.3	-3.8	-2.6	2.65
2000.....	6.4	4.2	-0.4	-0.9	2.2	+4.2	+2.0	-2.6	-3.1	2.70
1000.....	12.5	12.0	5.7	6.7	8.1	+4.4	+3.9	-2.4	-1.4	2.51
0.....	16.8	17.3	13.5	13.4	14.1	+2.7	+3.2	-0.5	-0.7	1.48
Mean from 0 to 4000 meters, 2.43										

TABLE 11.—*Adopted mean temperatures and gradients on the 1000-meter levels for American and European cyclones and anticyclones.*

Height in meters.	Winter.				Summer.			
	American.		European.		American.		European.	
	T_0	ΔT 1000	T_0	ΔT 1000	T_0	ΔT 1000	T_0	ΔT 1000
	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$
16000	-73.9	-3.5	-75.4	-3.5	-72.2	-4.0	-71.1	-3.0
15000	-70.4	-3.0	-71.9	-3.0	-68.2	-3.0	-68.1	-2.5
14000	-67.4	-2.5	-68.9	-2.5	-65.2	-3.0	-65.6	-2.5
13000	-64.9	-2.0	-66.4	-2.0	-62.2	-4.0	-63.1	-4.0
12000	-62.9	-4.0	-64.4	-4.0	-58.2	-5.5	-59.1	-5.5
11000	-58.9	-5.5	-60.4	-5.5	-52.7	-6.0	-63.6	-6.5
10000	-53.4	-6.5	-54.9	-6.5	-46.7	-6.7	-47.1	-7.0
9000	-46.9	-7.1	-43.4	-7.0	-40.0	-7.0	-40.1	-7.5
8000	-39.8	-7.0	-41.4	-7.0	-33.0	-6.5	-32.6	-7.0
7000	-32.8	-6.8	-34.4	-7.0	-25.5	-6.9	-25.6	-6.5
6000	-26.0	-6.5	-27.4	-7.0	-20.6	-6.5	-19.1	-6.1
5000	-19.5	-5.5	-20.4	-5.9	-14.1	-7.0	-13.0	-5.8
4000	-14.0	-5.0	-14.5	-5.5	-7.1	-7.5	-7.2	-5.1
3000	-9.0	-4.7	-9.0	-4.9	+0.4	-6.0	-2.1	-4.3
2000	-4.3	-2.8	-4.1	-4.4	+6.4	-5.4	+2.2	-5.9
1000	-1.5	-4.3	+0.3	-4.0	+11.8	-5.4	+8.1	-6.0
000	+2.8		+4.3		+17.2		+14.1	

and -14.5° , and for summer -7.1° and -7.2° , respectively, for America and Europe. This indicates that, after escaping from the confusion in the circulation near the surface, about

the same temperatures prevail in the midst of the track of the cyclones and anticyclones in the Northern Hemisphere, that is over Blue Hill, latitude 42° , in the United States, and over Hald, latitude 56° , and Berlin, latitude 52° , in Europe, though the stations are themselves in different latitudes. We can, therefore, admit the European data to apply to the American temperatures above the 4000-meter level, at least approximately.

I have also, in Table 11, extended the temperatures and gradients up to 16,000 meters, using the known temperatures derived from European observations so far as possible in these high elevations, because I wish to secure some tentative values of the pressure, the density, and the gas factor at those altitudes, to be computed by the formula which will be introduced in the following paper of this series. It should be observed that the gradients are smaller in the lower levels than in the middle levels, 4000 to 10,000 meters, and that they are again smaller in the high levels above 10,000 meters. This type of variation in the gradients conforms to the warm zone observed by Teisserenc de Bort at the high levels, his "isothermal" zone, so that there are in the atmosphere two zones of relatively small gradients, the lower near the surface, from 0 to 4000 meters, and the higher above 10,000 meters. In these zones there are rapid alternations in temperature, as compared with the middle zone, showing that mixing currents prevail in these two strata of the atmosphere, the lower, near the surface, being the region where the cyclones and anticyclones are generated by the counterflow of warm and cold currents, and the latter, in the high levels, where the warm air overflowing from the Tropics mixes with cold air from the polar zones. As this is a subject belonging to the general circulation, further discussion of it will be postponed to a later paper. It should also be noted that I have gradually eliminated the annual variation in the temperature in the levels above 4000 meters, which is, I suppose, in agreement with the observed temperatures in the high levels. These adopted data will at least serve as examples, approximate to the truth, for introduction into the thermodynamic discussions to follow.

THE VARIATIONS OF TEMPERATURE IN THE SEVERAL SECTORS OF CYCLONES AND ANTICYCLONES ON EACH 1000-METER LEVEL.

Having derived from numerous direct observations the approximate normal temperatures prevailing in typical cyclones and anticyclones, I proceed to deduce a somewhat better average system of temperatures by the following discussion:

If T_0 is the mean or undisturbed temperature which prevails at a given level, independent of any local cyclonic disturbance, and T the temperature at a given disturbed point, as in the several sectors of the high and low areas, then $T_0 = T + \Delta T$, and $-\Delta T = T - T_0$ is the correction to transpose the mean T_0 into the observed T . Thus, in Table 9, under American winter high areas, at the north sector of the 4000-meter level, the temperature is -7.9° , which is $+6.1^{\circ}$ warmer than the mean temperature of that level -14.0° , taken over the combined high and low areas of the region. The variations ΔT of the second section of Tables 9 and 10 are, therefore, the corrections to change the mean T_0 to the observed temperature T , in which the sign $+$ means warmer, and the sign $-$ colder than the average temperature T_0 . An inspection of these tables will show the prevailing temperature variations in the several quadrants, and a comparison of the American with the European variations up to 4000 meters shows the extent to which the data fail to conform precisely. Allowance must, of course, be made for the fact that Blue Hill is near the ocean on the eastern side of a great continent, and Hald and Berlin on the northwestern side of another continent, so that the permanent masses of air, as the high areas to the northwest of Blue Hill in winter, but to the northeast of Hald and Berlin, or the high area to the southeast of Blue Hill in summer, and to the southwest of Hald and Berlin, must necessarily make

differences as to the temperature configurations of the several quadrants in the American and European systems. We have been proceeding on the assumption that the northern sector of the cyclone contains the "saddle" of pressure described in my papers on the barometric distribution, and while I have found it actually oriented in any one of the four quadrants at different times, yet it is situated to the north of the center in the great majority of instances for the United States.

If the mean variation of ΔT is computed up to 4000 meters, without regard to sign, to show the average relative distortion in the temperature level surfaces, we find for the—

American winter areas, the mean $\Delta T = 3.14$,
 American summer areas, the mean $\Delta T = 2.00$,
 European winter areas, the mean $\Delta T = 2.21$,
 European summer areas, the mean $\Delta T = 2.43$.

The American winter disturbance is 1.57 times that of summer; the European winter disturbance is 0.91 time that of the summer; the American winter disturbance is 1.42 times that of Europe; and the American summer disturbance is 0.82 time that of Europe. On the whole the disturbances of temperature were about the same in each of these districts in the weather conditions when the direct observations made in balloon and kite ascensions were executed. What the relative values are in more strongly developed storms remains to be determined by further investigations.

In order to construct the mean temperature variations, ΔT , adopted in Table 12, we proceeded as follows: The variations of Tables 9 and 10 were plotted on diagrams like fig. 5, the American up to 4000 meters, the European up to 6000 meters, and then lines dividing the warm areas from the cold areas were drawn, the result showing that there is a similar configuration in spite of minor differences. Then the mean values of these two systems were taken, that is, the arithmetical means of the several pairs of American and European values in the same sector, so that the two observed systems were reduced to one system, which made another drawing like fig. 5. The sequence of the numbers in the same sector from the ground up through the several levels, such as appear in Table 12, was then examined, and, on the assumption that the change from one level to another should be gradual, and in harmony with those above and below, it was possible to detect imperfections in the observational data. There were not many important changes necessary to be made, and the final changes can be checked by the reader from Table 13, which is the adopted mean system taken by averaging Tables 9 and 10, together with the occasional minor corrections mentioned. These gradient variations, that is, the ΔT of Table 12, can be checked in the following way: The sum of the variations, ΔT , found on the same level, as 10,000, 9000, etc., should be zero, if the adjustments are relatively perfect. An examination of the check sums shows that they conform sufficiently for our purposes, as already defined. The vertical sequence of the numbers in the several sectors, N., E., S., W., of the high and low areas, winter and summer, now harmonizes so far as minor irregularities in our observational data are concerned, but it will be proper to modify this adopted system whenever suitable evidence of any important inaccuracy can be procured.

We may observe that, generally, the north and west sectors of the winter high areas are warm throughout, the east is cool in the high, and warm in the low, while an inversion of temperature takes place in the south sector. The last sector, to some extent, conforms to Professor Hann's inversion data, but the other sectors do not confirm his view. In the winter low areas the north and east sectors are warm, the west is cool, and the south again inverts. In the summer high areas the east sector is cool, the west is warm, and the north is warm up to 7000 meters, while the south inverts at 5000 meters from cool to warm; in the low areas the north and east sectors are warm, and the south and west sectors are generally cool.

TABLE 12.—*Adopted mean temperature variations, $-\Delta T$, in American and European cyclones and anticyclones.*

WINTER.												
Height in meters.	Mean temperature.		High areas. $T - T_0 = -\Delta T$				Low areas. $T - T_0 = -\Delta T$				Check sums.	
			$T_0 \quad -\Delta T$				$T_0 \quad -\Delta T$					
	$^{\circ}C.$	$^{\circ}F.$	N.	E.	S.	W.	N.	E.	S.	W.	$^{\circ}C.$	$^{\circ}F.$
10000....	-54.2	-6.5	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$	$^{\circ}C.$
			-1.0	-2.0	+0.5	+2.0	+2.2	+3.0	-2.0	-2.8	+7.7	-7.8
9000....	-47.7	-7.1	-0.5	-2.5	+0.9	+2.4	+2.0	+3.0	-2.2	-3.0	+8.3	-8.2
8000....	-40.6	-7.0	+0.0	-3.0	+1.3	+3.0	+1.5	+2.7	-2.5	-3.3	+8.5	-8.8
7000....	-33.6	-6.9	+0.7	-3.5	+1.5	+4.0	+1.2	+2.5	-2.6	-3.5	+9.9	-9.6
6000....	-26.7	-6.7	+1.4	-4.0	+1.2	+5.0	+1.1	+2.2	-2.8	-3.8	+10.9	-10.6
5000....	-20.0	-5.7	+2.0	-4.6	0.0	+5.7	+1.2	+2.0	-2.8	-4.0	+10.9	-11.4
4000....	-14.3	-5.3	+2.8	-4.3	-1.2	+5.6	+1.8	+1.8	-2.4	-3.5	+12.0	-11.4
3000....	-9.0	-4.8	+2.7	-4.0	-1.3	+4.8	+2.2	+1.4	-2.3	-3.4	+11.1	-11.0
2000....	-4.2	-3.6	+2.0	-3.8	-2.7	+4.0	+2.4	+1.2	-1.3	-3.0	+9.6	-10.8
1000....	-0.6	-4.2	+1.5	-3.1	-2.5	+3.3	+1.7	+1.3	+1.1	-2.2	+8.9	-7.8
0....	+3.6		+0.6	-2.1	-3.5	+2.0	+0.9	+1.1	+2.9	-1.7	+7.5	-7.3
SUMMER.												
10000....	-46.9	-6.8	-1.2	-1.5	+0.2	+1.0	+4.8	+1.2	-2.0	-2.5	+7.2	-7.2
9000....	-40.1	-7.3	-0.6	-2.0	+0.5	+1.5	+4.6	+1.2	-2.3	-2.8	+7.8	-7.7
8000....	-32.8	-6.7	-0.0	-2.5	+1.0	+1.8	+4.4	+1.0	-2.7	-3.0	+8.2	-8.2
7000....	-26.1	-6.2	+0.7	-3.0	+1.5	+2.0	+4.2	+1.2	-3.0	-3.2	+9.6	-9.2
6000....	-19.9	-6.3	+1.4	-3.3	+1.2	+2.4	+4.0	+1.4	-3.0	-3.5	+10.4	-9.8
5000....	-13.6	-6.4	+1.9	-3.2	-0.1	+2.8	+3.3	+1.4	-2.7	-3.0	+9.4	-9.0
4000....	-7.2	-6.3	+2.3	-3.2	-0.5	+2.2	+2.8	+1.3	-2.0	-2.6	+8.6	-8.3
3000....	-0.9	-5.2	+1.9	-2.9	-0.4	+2.0	+2.4	+0.3	-1.0	-2.6	+6.6	-6.9
2000....	+4.3	-5.7	+1.4	-2.2	-0.4	+1.6	+1.9	+0.2	0.0	-2.4	+5.1	-5.0
1000....	+10.0	-5.7	+0.5	-1.9	-1.2	+1.0	+1.2	+0.4	+1.0	-2.2	+4.1	-5.3
0....	+15.7		+0.2	-1.6	-2.0	+0.4	+0.8	+0.8	+2.8	-1.5	+5.0	-5.1

These facts can be seen plainly on fig. 5, which reproduces the numbers of Table 12 in a graphic form. I have added a few arrows to suggest the probable flow of the cool and warm currents in the several levels. Thus, in the 10,000-meter level the cool current is from the northwest, in the middle levels, as 5000 meters, from the north, and in the lower levels from the northeast. The warm current is generally from the south with a smaller rotation from the southwest through the south to the southeast. I should like, in this connection, to call special attention to figs. 6 and 7 of my paper III: "The observed circulation of the atmosphere in the high and low areas," MONTHLY WEATHER REVIEW, March, 1902, where the results of the work of the Weather Bureau on the circulation during the International Cloud Year, 1896-7, were summarized. The observed vectors and the components which disturb the normal motion of the atmosphere are there given. Two characteristic features were then marked with the letter A, one on the northern quadrants of the cyclonic components, and the other on the southern quadrants of the anticyclonic components. By comparing with fig. 5 of this paper, it will be seen that there is a rotation through about 90° , from the northwest to the northeast in the north quadrant, both for velocity components and for temperature variations in passing from the 10,000-meter levels to the surface; and that the rotation in the south quadrant is less in the respective cases. It is proper to infer that the two systems are mutually interdependent, and that the movement of the circulation is attended by a corresponding variation in the temperature distribution.

At the time of constructing the chart of the velocity vectors, I was unable to explain its meaning, and as it differed radically from the charts of velocity-motion published by Mr.

Clayton for the Blue Hill observations, and by Professor Hildebrandsson for European observations, it was desirable that some criterion should be found which would indicate which configuration is to be accepted. My charts, as there explained, depended upon an asymmetric temperature structure of the cyclone and anticyclone, while the Clayton-Hildebrandsson charts seemed to favor a symmetrical system, such as Ferrel had assumed to exist in his theoretical discussions. The agreement, however, of these two independent sets of observations, velocity vectors by the Weather Bureau, and temperatures by Blue Hill, Hald and Berlin, would seem to indicate that the asymmetric system of currents must be adopted. While I can not claim that the numerical values of Table 12, and fig. 5, of this paper are entirely correct, it yet seems probable that the prevailing type of the temperature configurations in the several levels has been made out with sufficient definiteness to permit a numerical discussion of the data, with the aid of the thermodynamic equations in connection with the general dynamic equations of motion, such as will be attempted in the other papers of this series.

THE MEAN TEMPERATURE, T , IN CYCLONES AND ANTICYCLONES.

By means of the variations of temperature, ΔT , given in Table 12, using the mean temperatures of the second column, we can find the mean temperatures in each sector, N., E., S., W., of the high and low areas for summer and winter. The result is given in Table 13, mean temperatures, T , in cyclones and anticyclones. The same data are plotted on fig. 6, and lines dividing the warm and cold areas are drawn, which are similar to those in fig. 5. It is not necessary to make any extended remarks about the distribution of temperatures at this point, but the relations can be illustrated by fig. 7, on which the temperatures of Table 13 are suitably plotted for each sector, the dotted lines standing for the warm areas and the full lines for the cool areas. The scale of the temperature abscissas is smaller than in figs. 3 and 4 of the preceding paper, in order to extend the elevation to 10,000 meters instead of 6000 meters.

From Table 13 we can obtain the temperature falls in 1000 meters for each sector by subtracting the successive temperatures, and such temperature falls per 1000 meters, or gradients, may be taken as the true average for the middle of the stratum. On fig. 8, marked the temperature fall per 1000 meters in each gradient, these results are plotted at the middle distances between the 1000-meter levels, and they are connected up by proper curves, which form an interesting study. The first remark is that the result of the observations in the east sector of the high levels in winter and summer probably need to be modified, because the east line has escaped the concentration common to all the other lines. The second point is that the curvature of the lines indicates the prevailing cloud strata; that is where the lines slope upward to the right, and where the air is relatively warmer than the mean gradient would admit, a cloud formation takes place, as cumulus in the 1000-2000-meter stratum, cirrus in the 8000-10,000-meter stratum, winter and summer, and alto-cumulus in the 4000-6000-meter stratum in summer. These levels contain an especially large number of mixing currents of warm and cold masses, and the latent heat of condensation of vapor to water adds a certain amount to the heat transported from other regions. The third and most important remark is the fact that the temperature gradients in the atmosphere are very variable from one level to another, and that they are always smaller than the adiabatic gradient, 9.87, in these latitudes. In the latitudes farther south and in the Tropics, at least in the lower strata, the adiabatic gradient is, on the other hand, often less than that actually observed in the atmosphere. Such relations of observed to adiabatic gradients will form the key to my treatment of the thermodynamic formulæ in their application to the atmosphere. It is apparent that the Ferrel Canal Theory

of the general circulation, poleward in the upper strata and equatorward in the lower strata, must be practically rejected or greatly modified in the following manner. The heat energy of the Tropics escapes toward the poles principally in two strata, the upper in the cirrus region, where there is mixing with cold currents from the poles, and the lower in the cumulus strata, where there is also mixing with the cold polar streams, while between these layers the intermediate strata are comparatively free from pronounced intermingling. The exact character and the extent of these operations in the cirrus level remain to be more fully investigated by suitable observations, which are required before the needed details can be secured. Especially in the tropical zones it is important to procure accurate thermal gradients up to high levels. The final solution of the problem of the circulation of the atmosphere is dependent upon such an exploration of the temperature conditions, because from these data alone can the corresponding pressure and density be computed.

TABLE 13.—Mean temperatures, T , in cyclones and anticyclones.
WINTER.

Height in meters.	Mean tempera- ture, T_0	High areas.				Low areas.			
		N.	E.	S.	W.	N.	E.	S.	W.
10000	° C. -54.2	° C. -55.2	° C. -56.2	° C. -53.7	° C. -52.2	° C. -52.0	° C. -51.2	° C. -56.2	° C. -57.0
9000	-47.7	-48.2	-50.2	-46.8	-45.3	-45.7	-44.7	-49.9	-50.7
8000	-40.6	-40.6	-43.6	-39.3	-37.6	-39.1	-37.9	-43.1	-43.9
7000	-33.6	-32.9	-37.1	-32.1	-29.6	-32.4	-31.1	-36.2	-37.1
6000	-26.7	-25.3	-30.7	-25.5	-21.7	-25.6	-24.5	-29.5	-30.5
5000	-20.0	-18.0	-24.6	-20.0	-14.3	-18.8	-18.0	-22.8	-24.0
4000	-14.3	-11.5	-18.6	-15.5	-8.7	-12.5	-12.5	-16.7	-17.8
3000	-9.0	-6.3	-13.0	-10.3	-4.2	-6.8	-7.6	-11.3	-12.4
2000	-4.2	-2.2	-8.0	-6.0	-0.2	-1.8	-3.0	-5.5	-7.2
1000	-0.6	+0.9	-3.7	-3.1	+2.7	+1.1	+0.7	+0.5	-2.8
0	+3.6	+4.2	+1.5	+0.1	+5.6	+4.5	+4.7	+6.5	+1.9

SUMMER.

10000	-46.9	-48.1	-48.4	-46.7	-45.9	-42.1	-45.7	-48.9	-49.4
9000	-40.1	-40.7	-42.1	-39.6	-38.6	-35.5	-38.9	-42.4	-42.9
8000	-32.8	-32.8	-35.3	-31.8	-31.0	-28.4	-31.8	-35.5	-35.8
7000	-26.1	-25.4	-29.1	-24.6	-24.1	-21.9	-24.9	-29.1	-29.3
6000	-19.9	-18.5	-23.2	-18.7	-17.5	-15.9	-18.5	-22.9	-23.4
5000	-13.6	-11.7	-16.8	-13.7	-10.8	-10.3	-12.2	-16.3	-16.6
4000	-7.2	-4.9	-10.4	-7.7	-5.0	-4.4	-5.9	-9.2	-9.8
3000	-0.9	+1.0	-3.8	-1.3	+1.1	+1.5	-0.6	-1.9	-3.5
2000	+4.3	+5.7	+2.1	+3.9	+5.9	+6.2	+4.5	+4.3	+1.9
1000	+10.0	+10.5	+8.1	+8.8	+11.0	+11.2	+10.4	+11.0	+7.8
0	+15.7	+15.9	+14.1	+13.7	+16.1	+16.5	+16.5	+18.5	+14.2

RESUME OF THE TYPICAL DISTRIBUTIONS OF THE VELOCITY, TEMPERATURE, AND PRESSURE IN CYCLONES AND ANTICYCLONES.

Having described the typical distribution of the velocity in the local circulations, as in the International Cloud Report, 1898, or the MONTHLY WEATHER REVIEW for March, 1902, the pressure distribution, as in the MONTHLY WEATHER REVIEW for February, 1903, and the temperature in this paper, it will be proper to summarize the results schematically, as in figs. 9 and 10. The former gives the resultants of the general circulation in the north temperate latitudes plus the local component disturbances, and the latter simply the components by themselves, separated from the prevailing eastward drift. In fig. 9 the velocity is seen to diverge more and more from the eastward direction in the higher levels, and gradually to break up into the well-known cyclonic and anticyclonic gyrations in the lower

Fig. 5.—Mean Temperature Variations, ΔT , in American and European Cyclones and Anticyclones.

Fig. 6.—Mean Temperature Distribution, T , in Cyclones and Anticyclones.

XXXIV—39. FIG. 7.—Adopted Temperature Distribution, Derived from the Mean American and European Observations.

FIG. 9.—Typical Distribution of the Velocity, Temperature, and Pressure in Cyclones and Anticyclones.

Fig. 10.—Typical Distributions of the Cyclonic and Anticyclonic Components of Velocity, Temperature, and Pressure.

levels. The anticyclone has a system of outward components from top to bottom, and the cyclone a system of inward components from bottom to top, but in neither case can there be any true inversion in the type of the system. The temperatures show that the wave motion is intensified on approaching the surface, as the strong eastward drift is gradually diminished in the lower levels. The pressure, on descending from one level to the other, in the same way gradually takes on the well-known features of the high and low pressure areas, the high areas standing with the "saddle" toward the south, and the low areas with the "saddle" toward the north. The *closed* isobars decrease in density from the surface upward, and disappear at two or three miles above the ground, being depleted at the top by penetration into the eastward drift. Whenever closed isobars occur there is a vertical component of the circulation, downward in anticyclones, upward in cyclones. There is evidently very little vertical movement in the upper levels of the atmosphere, where the isobars are mere wavy lines, unless some unobserved closed isobars occur, as is probably the case in the development of hurricanes in the Tropics.

In fig. 10 the disturbing components are given for the velocity, temperature, and pressure. In the velocity of the anticyclone there is a gradual transition of the known outflowing structure at the surface into a simple loop in the upper levels, the orientation being changed only a little; in the

cyclone the inflowing components are better preserved from the surface to the higher levels, but there is a distinct rotation of the structure through about one quadrant. The temperatures show the maximum disturbances on the boundary of the high and low areas, with a distinct rotation of both the cold and warm areas through one quadrant. The pressure disturbances consist of closed isobars gradually diminishing into loops in the higher levels and rotating through one quadrant, especially in the cyclone. In one aspect the analytical solution of this dynamic structure is simpler than that demanded in Ferrel's or in Guldberg and Mohn's adopted types of vortices, but it is certainly different from either of them. It is evidently necessary to distinguish carefully between the cyclonic system proper and the resultant system formed by its combination with the general eastward drift, so that the mathematical analysis shall not deal with the components and resultants indiscriminantly. It is not proper to appeal to observed resultant motions in the atmosphere in verification of a theory applying solely to the components, namely, the cyclonic and anticyclonic gyrations as examples of a special form of vortex. Having thus found at least an approximate system of correlated velocities, temperatures, and pressures in the atmosphere, it will be possible to approach the mathematical analysis of the structure with some prospect of a satisfactory solution.

STUDIES ON THE THERMODYNAMICS OF THE ATMOSPHERE.

By Prof. FRANK H. BIGELOW.

III.—APPLICATION OF THE THERMODYNAMIC FORMULÆ TO THE NONADIABATIC ATMOSPHERE.

THE NONADIABATIC ATMOSPHERE.

In the preceding papers of this series it has been shown that in the latitudes of the temperate zones the atmosphere is not arranged in such a way that the thermal gradients conform to the adiabatic rate of change along the vertical, $\frac{dT}{dz} = 9.867^{\circ}\text{C. per 1000 meters}$, but that they depart from that rate, being generally much less. In the tropical zones the few available observations indicate that in the lower strata the temperature gradient exceeds that amount, or is equal to it. Thus O. L. Fassig¹ found the mean of four ascents at Nassau, in June-July, 1904, to be 28.3°C. at the surface and 18.3°C. at 1000 meters, evidently the adiabatic rate. H. Hergesell² found for 16 ascents on the Atlantic, in the region between the African coast, the Canaries, and the Azores, the following temperatures:

Height.	<i>T</i>	ΔT	
Meters.	$^{\circ}\text{C.}$	$^{\circ}\text{C.}$	
5000	(-10.0)	- 8.5	Adiabatic gradient,
4000	- 1.5	-10.5	
3000	9.0	- 9.0	
2000	18.0	- 8.4	
1000	26.4	+ 3.4	
0	23.0		

This is an average adiabatic rate from the lower cloud level to 5000 meters, but differs widely from that rate from the surface to 1000 meters. He also reports³ an adiabatic rate, for the ascensions of 1905, from the surface to 1350 meters, then a zero or even a positive temperature gradient to 3550 meters, above that a rather rapid fall to 13,000 meters, and higher still in the atmosphere a slower rate, indicating an intrusion of warm air.

As the result of my kite work from the U. S. S. *Cæsar*, over the North Atlantic Ocean between Hampton Roads and Gibraltar, during the Spanish Eclipse Expedition, I found the temperatures as follows, for the dates June 24, 26, 28, 29, 30, July 5, and September 22, 1905:

Height.	Mean of 5 ascents,	July 5.	Sept. 22.
Meters.	$^{\circ}\text{C.}$	$^{\circ}\text{C.}$	$^{\circ}\text{C.}$
1000	16.9	7.9	15.6
800	17.1	9.3	17.9
600	17.6	11.1	18.5
400	18.5	13.2	14.6
200	19.6	15.6	17.8
0	22.1	18.0	20.9

These evidently approximate the adiabatic rate on July 5, but depart from it on the other dates, notably on September 22, when the kite ran through a warm stratification, probably blown from the peninsula of Spain over the ocean. These examples show plainly that meteorologists must be prepared to discuss the problems of the circulation of the atmosphere

¹ Kite flying in the Tropics. O. L. Fassig. M. W. R., December, 1903.
² Sur les ascensions de cerfs-volant exécutées sur la Méditerranée et sur l'océan Atlantique, 1904. H. Hergesell. Note in Comptes Rendus, Jan. 30, 1905.
³ Die Erforschung der freien Atmosphäre über dem Atlantischen Ocean, 1905. H. Hergesell. Met. Zeit. November, 1905.

whether the thermal vertical gradients are adiabatic or not, and since our common formulæ are confined to the adiabatic case, it is an important study to learn how they can be practically modified and rendered flexible enough to meet the actually existing conditions.

FIG. 11.

I have made an attempt to indicate the probable arrangement of the isothermal surfaces in the earth's atmosphere by means of fig. 11. In the tropical zones the adiabatic rate prevails up to a certain height, as the dotted line, and above that a slower rate. In the temperate zones there is an intrusion of the adiabatic rate into the lower levels and a mixing area, but generally the temperature-fall is less than the adiabatic rate, resulting in a small gradient near the surface and up to 3000 meters, a more rapid fall to 10,000 meters, and again a slower fall due to a second intrusion of warm air from the Tropics. In the polar zones the temperature gradients are probably small, the air being generally cold, and having only small changes from the surface upward. These suggested isothermal lines should be compared with the circulation described in my paper, MONTHLY WEATHER REVIEW, January, 1904, fig. 19, where the results of this intrusion of the types I and II between the temperate and the tropical zones are explained. The arrows are reproduced on fig. 11, where it is seen that three circuits are proposed for each hemisphere; (1)

the tropic, circulating anticlockwise; (2) the temperate-tropic, circulating clockwise; and (3) the temperature-polar, circulating feebly anticlockwise for the Northern Hemisphere. In the temperate zones the local cyclonic and anticyclonic systems represent the products of the vertical as well as the horizontal mixing of the currents of air derived by transportation from different latitudes. The excess of heat of the Tropics, producing an adiabatic distribution of temperature in their lower strata, works out poleward at the top and at the bottom by irregular streams, which produce a varying system of temperature gradients in the atmosphere of the temperate zones, standing about midway in value, namely, 5.0° C. per 1000 meters, between that prevailing in the Tropics, 9.87° C. per 1000 meters, and that probably prevailing in the polar zones, as 2.0° to 3.0° C. per 1000 meters. The interchange of heat between the Tropics and the polar zones is by means of these three more or less irregular circuits, which produce primarily the well-known masses of permanent high or low pressure areas standing over the oceans and continents, and secondarily the rapidly migrating cyclonic gyrations of the temperate zones. We shall make an effort to approach our study of this complex circulation by a transformation of the thermodynamic formulæ into forms which will be suitable for computations in the actual atmosphere, as distinguished from an adiabatic but fictitious atmosphere, which has commonly been discussed by meteorologists.

DEVELOPMENT OF THE THERMODYNAMIC FORMULÆ.

In the formulæ derived for discussing the circulation of the atmosphere, it is important that the velocity should be expressed as a function of the temperature in a nonadiabatic atmosphere. It has been generally the custom to treat the velocity as a function of the pressure P , the density ρ , and the gravity g , but it will be equally valid and more valuable to make it a function of the temperature T , the specific heat at a constant pressure C_p , and the gravity g . We must in doing this assume the applicability of two physical laws in the atmosphere. There has been a difficulty in connecting the results obtained by these two methods, which will be pointed out in this paper and their reconciliation will be explained.

1. THE FIRST FORM OF THE BAROMETRIC FORMULÆ.

The special feature of this formula is that the density ρ is eliminated by the following process: Assume the Boyle-Gay-Lussac law, $P = \rho R T$, and the pressure law, $-dP = \rho g dz$,

$$(1) \text{ Then, } -\frac{dP}{P} R T = g dz.$$

Since $R = \frac{P_0}{\rho_0 T_0}$, for the standard conditions, we have,

$$(2) -\frac{dP}{P} \frac{P_0 T}{\rho_0 T_0} = g dz.$$

By definition $P_0 = B_0 \rho_m g_0$, and $\frac{dP}{P} = \frac{dB}{B}$, so that,

$$(3) -\frac{dB}{B} \cdot \frac{B_0 \rho_m g_0 T}{\rho_0 M T_0} = g dz, \text{ for common logs.}$$

For the hypsometric formula the gravity g is computed from the standard gravity g_0 by the factors, $(1+\gamma) = (1+0.0026 \cos 2\varphi)$ for latitude, and $\left(1 + 1.25 \frac{h}{R}\right) = (1+0.000000196)h$ for altitude, since $g_0 = g(1+\gamma) \left(1 + 1.25 \frac{h}{R}\right)$.

In integrating for an atmosphere composed of dry and moist air between the heights z_0 and z , the temperature term T , which is variable, is taken as the mean temperature of the air column $z-z_0$, and the moist air is accounted for by the factor, $(1+\beta) = \left(1 + 0.378 \frac{e}{B}\right)$, where e is the vapor tension. Hence, the inte-

gral mean temperature is,

$$\frac{1}{z-z_0} \sum_{z_0}^z T_z = T_m, \text{ and } \frac{T_m}{T_0} = (1 + 0.367\theta) = (1 + a\theta).$$

We must pass from $\frac{P_0}{P} = \frac{B_0}{B} \left(1 + 1.25 \frac{h-h_0}{R}\right)$, to logarithms, $\log \frac{P_0}{P} = (1+0.00157) \log \frac{B_0}{B} = (1+\gamma) \log \frac{B_0}{B}$, by adding the factor $(1+\gamma)$.

$$\text{Finally, } \frac{B_0 \rho_m}{\rho_0 M} = K = 18400,$$

$$\left. \begin{array}{l} \text{for } B_0 = 0.760 \text{ meter} \\ \rho_m = 13595.8 \\ \rho_0 = 1.29305 \\ M = 0.43429 \end{array} \right\} \text{ in the meter-kilogram system.}$$

Hence, by integration,

$$(4) -\int_{z_0}^z \frac{dB}{B} \cdot \frac{B_0 \rho_m g_0}{M \rho_0} \cdot \frac{T (1 + 0.378 \frac{e}{B})}{T_0} = \int_{z_0}^z g dz, \text{ because,}$$

$$(5) \log \frac{B_0}{B} \cdot \frac{Kg_0}{T_0} T_m (1 + 0.378 \frac{e}{B}) = g_m (z - z_0).$$

If $\frac{Kg_0}{T_0} = K_1$ is computed as a new barometric constant, and

$$T_m (1 + 0.378 \frac{e}{B}) = T_r, \text{ the virtual temperature, then,}$$

$$(6) K_1 T_r \log \frac{B_0}{B} = g_m (z - z_0) \text{ in mechanical units.}$$

I have computed the logarithmic tables 91, 92, 93 of the International Cloud Report, 1898, in such a form that the dry air temperature term m , the humidity term βm , and the gravity term γm , are kept separate from each other in

$$(7) \log B_0 = \log B + m - m\beta - m\gamma,$$

for the sake of accurate and flexible applications in all possible meteorological computations. Auxiliary tables can be constructed from these primary tables for any desired applications, by way of shortening the work in special cases, such as in numerous reductions to any selected plane, or in computing the pressures from point to point in the atmosphere, using as arguments the temperatures and humidities observed in balloon or kite ascensions. Especially, they can be used to compute the dynamical units of force, or work required to pass from point to point, by simply extracting $(z-z_0)$ from the tables, with the temperature, humidity, and pressure as the arguments and multiplying $(z-z_0)$ by g_m , so that

$$(8) -\int_{z_0}^z \frac{dP}{P} = g_m (z - z_0),$$

when there is no circulation or velocity term, $\frac{1}{2}(q^2 - q_0^2)$. This result is in conformity with the equation,

$$-dP = \rho g dz,$$

with which we began this discussion.

II. THE SECOND FORM OF THE BAROMETRIC FORMULÆ IN AN ADIABATIC ATMOSPHERE.

In formula (108a) of my collection in the International Cloud Report the abnormal form for dry air was written:

$$(9) \frac{P}{P_0} = \left(\frac{T_0 - \theta_m h}{T_0} \right)^m, \text{ which is}$$

$$\frac{B}{B_0} = \left(\frac{T_0 - \frac{dT}{dh} h}{T_0} \right)^m = \left(\frac{T}{T_0} \right)^m,$$

where $-\frac{dT}{dh}$ is the actual vertical gradient of temperature and

the exponent is undetermined. We acquire from the observations a vertical gradient, $-\frac{dT}{dh}$, which generally differs from the adiabatic gradient, $-\frac{dT_a}{dh}$, and seek to determine the proper value of the exponent m .

From my formula (73), in the adiabatic state for $dQ = 0$, we have $0 = C_p dT - RT \frac{dP}{P}$, in mechanical units.

(10) Hence, $\frac{dP}{P} = \frac{C_p}{R} \frac{dT}{T}$, and integrating,

(11) $\log \frac{P}{P_0} = \frac{C_p}{R} \log \left(\frac{T}{T_0} \right)$, or $\frac{P}{P_0} = \left(\frac{T}{T_0} \right)^{\frac{C_p}{R} = \frac{k}{k-1}}$

(12) Again, $\frac{C_p}{R} = -g \frac{dz}{dT} \cdot \frac{T_0}{T_0 g_0} = -g \frac{dz}{dT} \cdot T_0 \cdot \frac{\rho_0}{P_0}$, and

(13) $\frac{C_p}{R} = -\frac{dz}{dT} T_0 \frac{g \rho_0}{g_0 \rho_m B_0}$. Substituting in (10)

(14) $\frac{dB}{B} = -\frac{dz}{dT} \cdot T_0 \frac{g}{g_0} \frac{\rho_0}{\rho_m B_0} \cdot \frac{dT}{T}$. Hence,

(15) $\int_{z_0}^z g dz = -\frac{B_0 \rho_m g_0}{M \rho_0 T_0} \frac{1}{z} \int_{z_0}^z dT \frac{dB}{B}$, for $T = \frac{1}{z} \int_{z_0}^z dT$,

as before. Hence, we see that $\frac{C_p}{R}$ supplies the constants for the barometric constant K in the adiabatic case only. These substitutions, (12), (13), can be verified by referring to the formulæ of Table 14. It is well known that the use of the

formula $\frac{B}{B_0} = \left(\frac{T}{T_0} \right)^{\frac{k}{k-1}}$ is not applicable in the actual atmosphere, except to give what is called by von Bezold the potential temperature T_p corresponding with (B, T) when reduced to the standard pressure B_0 .

Making the following substitutions,
 $B_0 \rho_m g_0 = P_0$, $\frac{P_0}{\rho_0 T_0} = R$, and $\frac{1}{z} \int_{z_0}^z dT = T_m$, we have

(16) $g(z-z_0) = R T_m \log \frac{P_0}{P}$ in Napierian logarithms, and

(17) $\log \frac{P_0}{P} = \frac{g}{R} \frac{z-z_0}{T_m}$, or $\frac{P_0}{P} = e^{\frac{g}{R} \frac{z-z_0}{T_m}}$

This result can be obtained again by another process.

Assume, $-dP = +g \rho dz$, and $\rho = \frac{P}{RT}$,

Then, $-\frac{dP}{P} = +\frac{g}{R} \frac{dz}{T}$, and by integrating,

(17)_a $\log \frac{P_0}{P} = \frac{g}{R} \int_{z_0}^z \frac{dz}{T} = \frac{g}{R} \cdot \frac{z-z_0}{T_m}$.

III. THE BAROMETRIC FORMULA IN A NONADIABATIC ATMOSPHERE.

In the preceding case it has been assumed that the temperature varies with the height by the adiabatic law, which is, $-\frac{dT_a}{dz} = \frac{g_0}{C_p} = \frac{1000}{P_0} = 0.0098695 \text{ }^\circ\text{C.}$, so that the temperatures

of the formulæ of section II, of which $\frac{P}{P_0} = \left(\frac{T}{T_0} \right)^{\frac{k}{k-1}}$ is the representative, must have this relation. Now it is known that this formula in the atmosphere does not apply, except in occasional instances, and we, therefore, shall seek a formula

of the same type which will admit other temperature gradients, $-\frac{dT}{dz}$, in a quasi-adiabatic atmosphere. It has been assumed that there was no addition or subtraction of heat in the variation of the pressures and temperatures, but as this is only a special case it will be proper to take the general case, where the quantity of heat dQ is added or subtracted, besides that acquired or lost during the expansion and contraction processes. Since in the stratifications of the atmosphere by currents possessing different thermodynamic properties, there is departure from the adiabatic state by the term dQ , we shall resume the full equation for discussion.

Fig. 12 will make our treatment clear.

FIG. 12.—The relation of the observed to the adiabatic gradient.
Let T_0 = the initial temperature at the height z_0
 T_a = adiabatic temperature at the height z
 T = observed temperature at the height z

Then the adiabatic gradient is $a_0 = -\frac{dT_a}{dz} = \frac{T_0 - T_a}{z - z_0}$,

and the observed gradient is, $a = -\frac{dT}{dz} = \frac{T_0 - T}{z - z_0}$.

Let the ratio of these gradients, $n = \frac{dT_a}{dT} = \frac{T_0 - T_a}{T_0 - T}$.

Having regard to the adiabatic thermodynamic equation,

(18) $0 = C_p dT_a - \frac{dP}{\rho}$

we observe that the thermal mass passes from (T_0, z_0) to (T_a, z) by the oblique path marked, T_0 to T_a in conformity with the formulæ just discussed; it can then be carried from the point T_a to the point T at the same level z by changing the temperature through $(T - T_a)$, and the addition of the heat

$Q = C_p (T - T_a)$. Now we have,

(19) $0 = C_p (T_a - T_0) - \int_{z_0}^z \frac{dP}{\rho}$, from (18), and adding,

(20) $Q = C_p (T - T_a)$, we obtain,

(21) $Q = C_p (T - T_0) - \int_{z_0}^z \frac{dP}{\rho}$, or in the differential form

(22) $dQ = C_p dT - \frac{dP}{\rho}$.

Since $dT_a = n dT$, we have $dQ = C_p (dT - dT_a) = C_p dT - C_p n dT$. Subtracting this value of dQ from equation (22) we find,

(23) $0 = C_p n dT - \frac{dP}{\rho}$, in a quasi-adiabatic form,

which is true in a stratum where n is constant, that is, where the gradient $-\frac{dT}{dz}$ is not changing.

Substituting, $\frac{1}{\rho} = \frac{RT}{P}$, we have,

(24) $0 = n C_p dT - RT \frac{dP}{P}$, and

Greater.		Adiabatic.		Less.	
$n = 0 = \frac{9.87}{\infty}$ $T_0 - T = \infty$		$n = 0.5 = \frac{9.87}{19.74}$ $T_0 - T = 19.74$		$n = 1 = \frac{9.87}{9.87}$ $T_0 - T = 9.87$	
		$n = 2 = \frac{9.87}{4.94}$ $T_0 - T = 4.94$		$n = \infty = \frac{9.87}{0}$ $T_0 - T = 0$	
$\frac{P}{P_0} = \left(\frac{T}{T + \infty}\right)^{0 \frac{k}{k-1}}$ $= \left(\frac{T}{T + \infty}\right)^0$ $= \left(\frac{1}{\infty}\right)^0 = 0$ $P = 0 \times P_0 = 0$		$\frac{P_{0.5}}{P_0} = \left(\frac{T}{T + 19.74}\right)^{0.5 \frac{k}{k-1}}$ $= \left(\frac{T}{T + 19.74}\right)^{1.73}$ $= A_{0.5} < A_1$ $P_{0.5} = A_{0.5} P_0 < P_1$		$\frac{P_1}{P_0} = \left(\frac{T}{T + 9.87}\right)^{1 \frac{k}{k-1}}$ $= \left(\frac{T}{T + 9.87}\right)^{3.46}$ $= A_1 < A$ $P_1 = A_1 P_0 < P_2$	
		$\frac{P_2}{P_0} = \left(\frac{T}{T + 4.94}\right)^{2 \frac{k}{k-1}}$ $= \left(\frac{T}{T + 4.94}\right)^{6.92}$ $= A_2 < 1$ $P_2 = P_0 A_2 < P_0$		$\frac{P_\infty}{P_0} = \left(\frac{T}{T}\right)^{\infty \frac{k}{k-1}}$ $= \left(\frac{T}{T}\right)^\infty$ $= 1^\infty = 1$ $P_\infty = P_0$	

FIG. 13.—The variations of the ratio $n = \frac{dT_a}{dT}$.

$$(25) \quad 0 = \frac{n C_p}{R} \frac{dT}{T} - \frac{dP}{P}, \quad \text{so that,}$$

$$(26) \quad \frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{n C_p}{R}} = \left(\frac{T}{T_0}\right)^{\frac{k}{k-1}} = \left(\frac{T}{T_0}\right)^{-\frac{g}{R} \frac{dz}{dT}} = \left(\frac{T}{T_0}\right)^g.$$

The last forms are found as follows:

By definition, $-\frac{dT_a}{dz} = a_0 = \frac{g}{C_p}$, and the ratio,

$$(27) \quad n = \frac{dT_a}{dT} = \frac{a_0}{a} = \frac{g}{C_p a} = \frac{g}{C_p} \frac{dz}{dT}. \quad \text{Since } \frac{k}{k-1} = \frac{C_p}{R},$$

$$n \frac{k}{k-1} = \frac{g}{C_p a} \frac{k}{k-1} = \frac{g}{R a} = -\frac{g}{R} \frac{dz}{dT}.$$

Our formula, therefore, differs from the adiabatic formula by the factor n in the exponent with $\frac{k}{k-1}$. This ratio n between the adiabatic and observed gradients, depends upon the amount of heat added or subtracted from an adiabatic atmosphere to produce the given observed atmosphere within the stratum $z-z_0$ where the gradient remains a constant. We can evidently pass from one stratum to an adjoining stratum either continuously by changing n gradually, or discontinuously by changing n abruptly. The ratio n is a new variable to be introduced into the thermodynamic equations in their application to the atmosphere, so that all the standard thermodynamic equations and discussions become available with this simple modification. Such an exposition as was given by M. Margules in his admirable paper, "Über die Energie der Stürme," which is limited to the adiabatic case, may be modified in this way and be made very useful in practical meteorology. It is rarely the case that computations of T_0 to T , from one level to another, z_0 to z , can be made by general dynamic formulæ, but they must usually be observed with balloons and kites.

The ratio, $n = \frac{dT_a}{dT} = \frac{\text{adiabatic gradient}}{\text{observed gradient}}$, can range between

the limits $n=0$ and $n=\infty$; for $n=1$ the gradient is adiabatic; for $n<1$ the cooling is more rapid than in the adiabatic gradient, as in summer afternoons when the ground is superheated and cumulus clouds are forming; for $n>1$ the cooling is less rapid than in the adiabatic gradient, as generally in the temperate and polar zones; the Tropics probably conform to the adiabatic gradient in the lower strata of the atmosphere.

IV. CONSTRUCTION OF THE PRIMARY DIFFERENTIAL EQUATION.

Under the assumption that n is variable we now differentiate the equation with the variables P, T, n ,

$$(28) \quad \frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{n \frac{k}{k-1}}.$$

Passing to logarithms,

$$(29) \quad \log \frac{P}{P_0} = n \frac{k}{k-1} \log \left(\frac{T}{T_0}\right), \quad \text{or for one limit,}$$

$$(30) \quad \log P = n \frac{k}{k-1} \log T. \quad \text{Differentiate,}$$

$$(31) \quad \frac{dP}{P} = n \frac{k}{k-1} \frac{dT}{T} + \frac{k}{k-1} \log T dn. \quad \text{Substitute } \frac{1}{P} = \frac{1}{\rho R T},$$

$$(32) \quad \frac{dP}{\rho R T} = n \frac{k}{k-1} \frac{dT}{T} + \frac{k}{k-1} \log T dn. \quad \text{Substitute } R \frac{k}{k-1} = C_p,$$

$$(33) \quad \frac{dP}{\rho} = n C_p dT + C_p T \log T dn. \quad \text{In common logs and to } dz,$$

$$(34) \quad \frac{dP}{\rho dz} = n C_p \frac{dT}{dz} + C_p T \log T \frac{dn}{dz}, \quad \text{for the vertical direction.}$$

Again, since $n C_p \frac{dT}{dz} = -g$, by this substitution we have,

$$(35) \quad \frac{dP}{\rho dz} = -g + C_p T \log T \frac{dn}{dz}, \quad \text{and hence,}$$

$$(36) \quad dP = -\rho g dz + \rho C_p T \log T dn.$$

We see then that the effect of the change from an adiabatic atmosphere to any other gradient is accomplished by adding the term $\rho C_p T \log T dn$.

If it should happen that besides the strictly mechanical velocities thus indicated there is a further expenditure of heat by radiation, it would be necessary to add the special term, $Q_0 - Q$, making, from (33),

$$(37) \quad \frac{P}{\rho} - \frac{P_0}{\rho_0} = (Q_0 - Q) + n C_p (T - T_0) + C_p T \log T (n - n_0).$$

It is better to say that the full term $C_p T \log T (n - n_0)$ has a radiation part, $(Q - Q_0)$, and a velocity part, $C_p T \log T (n - n_0)$.

The factor n , due to an addition or subtraction of heat other than by adiabatic expansion and contraction, fully accounts for the presence of a nonadiabatic gradient, through the stratification of the layers of air due to transportation horizontally from one latitude to another, or generally from one place to another; or else through the addition or subtraction of latent heat in the condensation of aqueous vapor to water, or by the

vaporization of water to aqueous vapor. In effect, by the practical use of the factor n , we can dispense with the difficult computations which occur in making an allowance for the action of the vapor contents of the atmosphere; or, on the other hand, we can substitute for n its equivalent in terms of such other computations as may be found convenient for particular purposes.

The corresponding formulæ involving P , T , R , ρ , and n , in terms of the temperature T , become,

(38) $\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{n k}{k-1}}$; $\log P = \log P_0 + \frac{n}{k-1} (\log T - \log T_0)$.

(39) $\frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{\frac{n}{k-1}}$; $\log \rho = \log \rho_0 + \frac{n}{k-1} (\log T - \log T_0)$.

(40) $\frac{R}{R_0} = \left(\frac{T}{T_0}\right)^{n-1}$; $\log R = \log R_0 + (n-1) (\log T - \log T_0)$.

(41) $\frac{\rho}{\rho_0} = \left(\frac{P}{P_0}\right)^{\frac{1}{k}}$; $\log \rho = \log \rho_0 + \frac{1}{k} (\log P - \log P_0)$.

It is evident that R is not constant except in the adiabatic system for $n = 1$; and that only that density determined through the use of n is generally valuable in the atmosphere.

V. APPLICATION TO THE GENERAL EQUATIONS OF MOTION.

We will now make the connection between this system of equations and the general equations of motion which have been employed in meteorology. From the equations (200) of the Cloud Report, we have, in connection with the differentiations of equation (37) along the axes x , y , z , for the acceleration,

(42)
$$\begin{cases} -\frac{1}{\rho} \frac{\partial P}{\partial x} = \frac{du}{dt} - \cos \theta (2\omega + \nu) v + \frac{uv}{r} \\ = \frac{\partial Q}{\partial x} - C_p n \frac{\partial T}{\partial x} - C_p T \log T \frac{\partial n}{\partial x} \\ -\frac{1}{\rho} \frac{\partial P}{\partial y} = \frac{dv}{dt} + \sin \theta (2\omega + \nu) w + \cos \theta (2\omega + \nu) u \\ = \frac{\partial Q}{\partial y} - C_p n \frac{\partial T}{\partial y} - C_p T \log T \frac{\partial n}{\partial y} \\ -\frac{1}{\rho} \frac{\partial P}{\partial z} = \frac{dw}{dt} - \sin \theta (2\omega + \nu) v - \frac{w^2}{r} + g \\ = \frac{\partial Q}{\partial z} - C_p n \frac{\partial T}{\partial z} - C_p T \log T \frac{\partial n}{\partial z} \end{cases}$$

Multiplying by dx , dy , dz , the equations for work are,

(43)
$$\begin{cases} -\frac{\partial P}{\rho} = u du - \cos \theta (2\omega + \nu) v dx + \frac{uv dx}{r} \\ = \partial Q - C_p n \partial T - C_p T \log T \partial n \\ -\frac{\partial P}{\rho} = v dv + \sin \theta (2\omega + \nu) w dy + \cos \theta (2\omega + \nu) u dy \\ = \partial Q - C_p n \partial T - C_p T \log T \partial n \\ -\frac{\partial P}{\rho} = w dw - \sin \theta (2\omega + \nu) v dz - \frac{uw dz}{r} + g dz \\ = \partial Q - C_p n \partial T - C_p T \log T \partial n \end{cases}$$

Since, by substituting $v dx = u dy$, $w dx = u dz$ and $w dy = v dz$, the terms in $(2\omega + \nu)$, the angular velocity of the earth and the atmosphere relative to it, disappear in the summation, they represent a deflecting force at right-angles to the direction of motion at the velocity q , which does not modify the circulation but only the path of motion. The integral, therefore, becomes between two places,

(44)
$$\begin{cases} \frac{P_0}{\rho_0} - \frac{P}{\rho} = \frac{1}{2} (q^2 - q_0^2) + g (z - z_0) \\ = Q - Q_0 - C_p n (T - T_0) - C_p T \log T (n - n_0) \end{cases}$$

It is noted that the term for the circulation $\frac{1}{2} (q^2 - q_0^2)$ must be added to the equations of sections I, II, III, IV, to pass from

the static state there considered to the circulating state here computed. Since we have

(45) $g (z - z_0) = -C_p n (T - T_0)$, it follows that

(46) $\frac{1}{2} (q^2 - q_0^2) = -C_p T \log T (n - n_0)$, for $Q - Q_0 = 0$, so that the circulation can be computed directly in terms of T and $(n - n_0)$. This proves that the energy of circulation is derived from the difference of temperature gradients in neighboring masses of air, where $n - n_0$ is not equal to zero. Moreover, since the integral of gdz around a closed curve is zero,

$\int_0 (gdz) ds = 0$, and we have the remaining,

(47)
$$-\int_0 \frac{dP}{\rho} ds = \int_0 \frac{dq}{dt} ds + \int_0 (gdz) ds = \int_0 q dq.$$

This is the equation employed by Bjerknes in his discussion of the circulation of the atmosphere, and is applicable only in closed curves, along all points of which P , ρ , q , or $q dq$ must be known by observations. The difficulty of securing such observed data simultaneously along the circuit at a given time is so great that this special case of the general equation will seldom be serviceable. In ordinary meteorology it is required to integrate between the two points, as in the same horizontal plane, or in a vertical direction. Since the term $\frac{1}{2} (q^2 - q_0^2)$ is expressed in mechanical measures and represents work done, then it may be taken as equivalent to $\frac{1}{2} (q^2 - q_0^2) = g (z' - z'_0)$, so that

(48) $\frac{1}{2} q^2 = g z'$, and $q^2 = 2 g z'$.

The circulation is therefore always equivalent to a falling velocity through the height z' , which may be computed.

Furthermore, since $Q - Q_0$ is also given in mechanical units, it may be taken as equivalent to

(49) $Q - Q_0 = g (z'' - z''_0)$, so that

(50) $Q = g z''$

and the stored up energy of radiation is equivalent to a vertical work.

It follows from these considerations that we obtain

(51)
$$\begin{cases} q \frac{dq}{dx} = \frac{dQ}{dx} - C_p n \frac{dT}{dx} - C_p T \log T \frac{dn}{dx}, \text{ in latitude.} \\ q \frac{dq}{dy} = \frac{dQ}{dy} - C_p n \frac{dT}{dy} - C_p T \log T \frac{dn}{dy}, \text{ in longitude.} \\ q \frac{dq}{dz} = \frac{dQ}{dz} - C_p T \log T \frac{dn}{dz}, \text{ in vertical.} \end{cases}$$

Since $P = B \rho_m g_0$, we obtain in a stratum of mean ρ ,

(52)
$$\begin{cases} (B_0 - B) = \frac{\rho}{2 g_0 \rho_m} [(q^2 - q_0^2) + 2 g (z - z_0)] \\ = \frac{\rho}{2 g_0 \rho_m} [(Q - Q_0) - C_p n (T - T_0) - C_p T \log T (n - n_0)] \end{cases}$$

It is readily perceived that the introduction of the factor n and the correlation of the pressure, velocity, gravity, radiation, specific heat, temperature, and gradient, in this double equation leads to an innumerable number of special combinations, taken in connection with the equations of thermodynamics. These embrace the first and second laws of thermodynamics, cyclic processes, the entropy S , the inner energy U , the thermodynamic potentials (F , ψ); the adiabatic, isodynamic, isometric, isothermal physical processes; differential relations with pairs of variables; thermodynamic surfaces and lines in gases; the adiabatic, isodynamic, isenergetic, and isopiestic processes with other variables in pairs; the gaseous, liquid, and solid phases; latent and specific heat; mixtures and chemical transformations, chemical dissociation, their solutions, and other relations, involving ionization, electrical and magnetic fields of force. This vast subject is open to meteorological investigation in the atmosphere, and will no doubt eventually lead to important practical results.

VI. FOUR SYSTEMS OF CONSTANTS FOR THE ATMOSPHERE.

In the application of these formulæ to computations of thermodynamic and dynamic problems in the atmosphere, it will be convenient to have for ready reference a table of the most important constants, with their equivalents in the four systems of units likely to be used. Table 14 presents such a compilation of constants in the following systems of mechanical or gravitational units:

1. Meter-kilogram-second-centigrade degrees.
2. Centimeter-gram-second-centigrade degrees.
3. Meter-gram-second-centigrade degrees.
4. Foot-pound-second-Fahrenheit degrees.

There is often so much confusion in discussing meteoro-

logical problems arising from the use of now one system, again another system, and even a hybrid system, that it may be a check against errors for those students who conform to the constants here given. The short formulæ in the first column define the quantities with precision, and the numerous transformations possible among them give rise to many combinations such as occur in various mathematical discussions. Indeed, it is surprising to note how large an amount of current meteorology, occurring in treatises and analytical papers, can be readily reduced to these elementary formulæ, and in reading a new presentation of primary principles it is proper to find whether they conform to these elementary theorems or not. We use the symbols:

TABLE 14.—Mechanical systems of constants for the atmosphere in gravitational units.

Formula.	S.	° C. Meter-kilogram.		° C. Centimeter-gram.		° C. Meter-gram.		° F. Foot-pound.	
			Log.		Log.		Log.		Log.
$P_0 = g_0 \rho_m B_n$	g_0	9.8060	0.99149	980.60	2.99149	9.8060	0.99149	32.172	1.50748
	ρ_m	13595.8	4.13340	13.5958	1.13340	13.5958	1.13340	846.728	2.92774
	B_n	0.760	9.88081	76.0	1.88081	0.760	9.88081	2.4934	0.39680
	P_0	101323.5	5.00571	1013235.	6.00571	101.3235	2.00571	67923.5	4.83202
$P_0 = g_0 \rho_0 l_0$	g_0	9.8060	0.99149	980.60	2.99149	9.8060	0.99149	32.172	1.50748
	ρ_0	1.29305	0.11162	0.00129305	7.11162	0.00129305	7.11162	0.080529	8.90595
	l_0	7991.04	3.90260	7991.04	5.90260	7991.04	3.90260	26217.3	4.41859
	P_0	101323.5	5.00571	1013235.	6.00571	101.3235	2.00571	67923.5	4.83202
$P_0 = R_0 T_0 \rho_0$	R_0	287.0334	2.45793	2870334.	6.45793	29.2712	1.46644	1716.43	3.23463
$= F \frac{C_p}{g_0}$	T_0	273.	2.43616	273.	2.43616	273.	2.43616	491.4	2.69144
$= -F \frac{dh}{dT}$	ρ_0	1.29305	0.11162	0.00129305	7.11162	0.0012935	7.11162	0.080529	8.90595
	P_0	101323.5	5.00571	1013235.	6.00571	101.3235	2.00571	67923.5	4.83202
$p_0 = \frac{P_0}{g_0}$	p_0	10332.8	4.01422	1033.28	3.01422	10.3328	1.01422	2111.23	3.32454
$C_p = \frac{k}{k-1} \frac{l_0}{T_0} g_0$	$\frac{k}{k-1}$	3.461545	0.53927	3.461545	0.53927	3.461545	0.53927	3.461545	0.53927
$= \frac{k}{k-1} R$	l_0	7991.04	3.90260	799104.	5.90260	7991.04	3.90260	26217.3	4.41859
$= \frac{g_0 l_0}{F}$	g_0	9.8060	0.99149	980.60	2.99149	9.8060	0.99149	32.172	1.50748
$= -g_0 \frac{dh}{dT}$	$\frac{1}{T_0}$	1	7.56384	1	7.56384	1	7.56384	1	7.30856
	C_p	993.5787	2.99720	9935787.	6.99720	993.5787	2.99720	5941.57	3.77390
$R_0 = \frac{l_0}{T_0} g_0$	l_0	7991.04	3.90260	799104.	5.90260	7991.04	3.90260	26217.3	4.41859
$= \frac{B_n \rho_m}{T_0 \rho_0} g_0$	g_0	9.8060	0.99149	980.60	2.99149	9.8060	0.99149	32.172	1.50748
$= \frac{P_0}{\rho_0 T_0}$	$\frac{1}{T_0}$	1	7.56384	1	7.56384	1	7.56384	1	7.30856
	R_0	287.0334	2.45793	2870334.	6.45793	287.0334	2.45793	1716.43	3.23463
$C_v = C_p - R_0$	C_v	706.5453	2.84914	7065453.	6.84914	706.5453	2.84914	4225.14	3.62584
$k = \frac{C_p}{C_v}$	k	1.4062486	0.14806	1.4062486	0.14806	1.4062486	0.14806	1.4062486	0.14806
	$k-1$	0.4062486	9.60879	0.4062486	9.60879	0.4062486	9.60879	0.4062486	9.60879
	$\frac{k}{k-1}$	3.461545	0.53927	3.461545	0.53927	3.461545	0.53927	3.461545	0.53927
	$\frac{1}{k-1}$	2.461545	0.39121	2.461545	0.39121	2.461545	0.39121	2.461545	0.39121
$-\frac{dT}{dh} = \frac{g_0}{C_p}$	$-\frac{dT}{dh}$	0.0098695	7.99429	0.00098695	5.99429	0.0098695	7.99429	0.0054147	7.73358
$\frac{1}{A_m} = \frac{g_0}{A}$	$\frac{1}{A_m}$	4185.57	3.62175	41855700.	7.62175	4185.57	3.62175	25027.7	4.39842
$A_m = \frac{A}{g_0}$	A_m	0.0002389	6.37829	2.389×10^{-8}	2.87829	0.0002389	6.37829	0.00003995	5.60158
$P'r. Th. U.$	Θ							3.962	0.59856
	F	1000.	3.00000	100	2.00000	1	0.00000	367.8	2.56560
	A	0.002343	7.36978	2.343×10^{-5}	5.36978	0.002343	7.36978	0.0012855	7.10906
	$\frac{1}{A}$	426.837	2.63022	42683.7	4.63022	426.837	2.63022	777.9	2.89094

P_0 = pressure in units of force, g_0 .
 ρ_0 = the weight of a given mass of atmosphere, $\rho_m B_n = \rho_0 l_0$.
 C_p = the specific heat at constant pressure.
 C_v = the specific heat at constant volume.
 $R = C_p - C_v, \quad k = \frac{C_p}{C_v}$
 $-\frac{dT}{dh}$ = the temperature fall per unit height in adiabatic state.
 $\frac{1}{A}$ = the mechanical equivalent of heat, 426.8 and 777.9.
 $\frac{g_0}{A}$ = the factor to change mechanical units to heat units.
 F = the factor connecting the thermal gradient and P_0 .
 θ = the number of British thermal units in 1 kilogram-degree.

VII. THE THERMODYNAMIC CONSTANTS FOR THE SUN.
There is much difficulty in passing from the thermodynamic conditions on the earth to the corresponding thermodynamic conditions on the sun. I have already approached this subject from the side of radiation in my "Eclipse Meteorology and Allied Problems," 1902, and from the method of Nipher's Formulæ, in my studies on the "Circulation of the Atmospheres of the Sun and of the Earth," 1904. I shall briefly present the same subject as the immediate development of the fundamental formulæ introduced in this paper. It is not so difficult to produce a self-consistent system of quantities as it is to find one which conforms to the actual physical state of the sun, and I conceive that it is proper to discuss this subject in several ways.

Specific heat.
From the preceding formulæ, we have,
(53) $-\frac{dT}{dz} = \frac{g_0}{C_p} = \frac{F}{P_0} = \frac{F}{g_0 \rho_m B_n} = \frac{F}{g_0 \rho_0 l_0}$. Hence,
(54) $C_p = \frac{\rho_m B_n}{F} \cdot g_0^2 = \frac{\rho_0 l_0}{F} \cdot g_0^2$.
Since $\rho_m B_n = \rho_0 l_0$ is a given mass, and F is constant for a given system of units, it follows that C_p is proportional to the square of the gravity. Taking the force of gravity on the sun,
(55) $(g)_{sun} = g_0 \times G = 9.806 \times 28.028 = 274.843$
it follows that the specific heat on the sun is
(56) $(C_p)_{sun} = C_p \times G^2 = 993.5787 \times (28.028)^2 = 780524$.

Adiabatic rate of temperature-fall.
(57) For the earth $-\frac{dT}{dz} = \frac{g_0}{C_p} = 9.8695^\circ$ per 1000 meters.
(58) For the sun $-\left(\frac{dT}{dz}\right)_{sun} = \frac{g_0 G}{C_p G^2} = \frac{9.8695^\circ}{28.028} = 0.32862^\circ$.
Mechanical equivalent of heat.
(59) From $-\frac{dT}{dz} = \frac{g_0}{C_p}$ for the earth, we have on the sun,
(60) $-\frac{dT}{Gdz} = \frac{g_0 G}{C_p G^2}$. Hence, by integration,

(61) $-C_p G^2 \int dT = g_0 G \int G dz$, or,
(62) $-C_p G^2 (T - T_0) = g_0 G (z - z_0)$.
If the change of temperature is 1° then,
(63) $-C_p G^2 = g_0 G (z - z_0) G$,
is the mechanical equivalent of heat, and is obtained by the fall of a mass through the height $(z - z_0) G$ under the force of gravity $g_0 G$. Whereas on the earth,
(64) $\frac{1}{A_m} = 4185.57 = 426.8 \times 9.8060$, we have
(65) $\left(\frac{1}{A_m}\right)_{sun} = 4185.57 \times (28.028)^2 = 3288046$.

Boyle-Gay-Lussac Law.
From the formulæ of Table 14, we have,
(66) $g_0 l_0 = \frac{P_0}{\rho_0} = R T_0 = C_p T_0 \frac{k-1}{k} = -g_0 \frac{dh}{dT} T_0 \frac{k-1}{k}$,
on the earth, and we infer that we shall have on the sun,
(67) $g_0 G \cdot l_0 G^2 = \frac{P_0 G^3}{\rho_0} = R G^2 \cdot T_0 G = C_p G^2 \cdot T_0 G$
 $= -g_0 G \cdot \frac{dh}{dT} G \cdot T_0 G \frac{k-1}{k}$. Hence,
(68) $l_0 G^2 = 7991.04 \times (28.028)^2$.
(69) $P_0 G^3 = 101323.5 \times (28.028)^2$.
(70) $R G^2 = 287.0334 \times (28.028)^2$.
(71) $T_0 G = 273^\circ \times 28.028 = 7652^\circ$,
if $\frac{k-1}{k}$ is retained a constant in both cases.

If the atmosphere of the sun is composed of some other material than ρ_0 of the earth's atmosphere, then the proper modification of the preceding quantities can be readily computed from terrestrial data.
Specific heat at constant volume.
For the earth, $C_v = C_p - R$, and hence, for the sun,
(72) $C_v G^2 = C_p G^2 - R G^2$
(73) $(C_v)_{sun} = C_v G^2 = 706.5453 \times (28.028)^2 = 555040$.
(74) Finally, $k = \frac{C_p \cdot G^2}{C_v \cdot G^2} = 1.4062486$, as a check.

This system throws the entire emphasis upon a change of gravity depending upon the mass of the central body, rather than upon the change of physical conditions implied in altering the ratio of the specific heats k . Since the temperature of the photosphere may in this way be taken as about 7652° , and the temperature gradient -0.32862° per 1000 meters, it follows that the effective temperature of radiation as determined by bolometer measures, 6100° , will be reached at the height of 4418 kilometers, or 2745 miles above the surface of the photosphere. This change of 1552° may be sufficient to meet the requirements of the spectroscopic observations in regard to the absorption and reversal of the spectrum lines. The gradient, -0.32862° per 1000 meters, is 28.028 times greater than that obtained by my other methods, the difference arising from the different distribution of the gravity factor G , which seems to be fully accounted for in these formulæ.

STUDIES ON THE THERMODYNAMICS OF THE ATMOSPHERE.

By Prof. FRANK H. BIGELOW.

IV.—NUMERICAL COMPUTATIONS IN THE VERTICAL ORDINATE.

THREE GENERAL THEORIES REGARDING THE FORMATION OF CYCLONES AND ANTICYCLONES.

There seem to be only three important general theories regarding the formation of cyclones and anticyclones in the earth's atmosphere, which may be referred to those authors who have been conspicuously associated with their mathematical developments: (1) Ferrel's cold center and warm center cyclones and anticyclones; (2) Oberbeck's symmetrical central cyclones and anticyclones; and (3) Bigelow's asymmetric cyclones and anticyclones. In my International Cloud Report, 1898, I reviewed the mathematical analyses of the first and second theories, and gave my reasons for thinking that they are inconsistent with the air currents as mapped out by the cloud observations, as well as with the distribution of temperature found in the lower strata. These theories start with the systems of isobars which near the surface are distributed symmetrically about a central axis, and they assume that the temperatures are similarly arranged, which is, however, not the case, as we know. The two central systems have their origin in the fact that the second equation of motion can be discussed in two ways. Thus, in the case of no friction, $k=0$, the equation

$$0 = \frac{dv}{dt} + (2n \cos \theta + v_1)u + kv,$$

can be integrated by introducing the idea of a boundary cylinder about the system at the radial distance ω_0 , whence is derived,

$$v = \left(\frac{2\omega^2}{\omega_0^2} - 1 \right) \omega n \cos \theta,$$

which is the tangential velocity at the distance ω . This is Ferrel's method and several difficulties regarding it are mentioned on page 615 of the Cloud Report. The second equation of motion can be given another form retaining the friction term, where $\lambda = 2n \cos \theta$, so that,

$$0 = \frac{\partial v}{\partial t} + \frac{uv}{\omega} + \lambda u + kv,$$

from which are derived two solutions,

$$\text{First } \begin{cases} u = -\frac{c}{2}\omega \\ v = +\frac{\lambda}{k-c} \cdot \frac{c}{2}\omega \end{cases}, \quad \text{Second } \begin{cases} u = -\frac{c}{\omega} \\ v = +\frac{\lambda}{k} \frac{c}{\omega} \end{cases}.$$

These form the basis of the theory developed by Guldberg and Mohn, Sprung, Oberbeck, Pockels, and others. My specific objections are summarized on page 623 of the Cloud Report.

The construction of a better theory was at that time very difficult, for two reasons, the first, that it involved breaking away from the large mass of current literature in meteorology, and that it introduced many new ideas concerning the general and the local circulations of the atmosphere, the two being intimately bound up together; the second, due to the lack of definite pressure and temperature observations in the higher strata of the atmosphere. In the course of chapters 8 and 11 of the Cloud Report the leading ideas regarding the asymmetric cyclone and anticyclone were sketched out, and a fairly clear idea was given of the probable truth regarding the formation of these circulating structures. Since that time the Weather Bureau has secured daily pressure maps for the United States on the three planes, sea level, 3,500 feet, and 10,000 feet, throughout an entire year, 1903. This valuable material has been carefully studied, and a report presented on the subject, with a summary of the results in the MONTHLY WEATHER RE-

VIEW, May, 1904. The recent publications of the temperature observations, made during balloon and kite ascensions in Europe and America, have in some degree supplied this deficiency, and we are, therefore, now trying to discuss more definitely the entire subject by means of these several data—velocity, temperature, and pressure—than has been possible heretofore. In the preceding papers of this series we have given the temperature data and the thermodynamic formulæ, and in this paper we shall confine our attention to formula (44) in the vertical ordinate.

COMPARISON OF THE NUMERICAL RESULTS OF COMPUTATIONS BY FORMULA (38) AND THE GENERAL BAROMETRIC FORMULA OF THE CLOUD REPORT (59).

Since we have introduced a new system of formulæ for the computation of the pressures, densities, and the gas factors, from the temperatures, through the use of the ratio n , the ratio of the adiabatic temperature gradient to the observed gradient, it will be desirable to compare the numerical results by some examples, showing the relation of the thermodynamic formulæ to the well-known barometric formulæ. Formula (59) can be written as follows:

$$z - z_0 = 18400 \left(\frac{273 + \theta}{273} \right) \left(\frac{B + .378e}{B} \right) \frac{g_0}{g} \left(1 + \frac{1.25 KM}{R} \right) \log \frac{B_0}{B}$$

In our new thermodynamic formula we have made no attempt to refine it by introducing corrections due to the vapor term, $\frac{B + .378e}{B}$, the gravity term, $\frac{g_0}{g}$, nor the land-mass term,

$\left(1 + \frac{1.25 KM}{R} \right)$. It is evident that these will require a small change in the value of n , and the subject is worth an investigation, but for our preliminary studies of cyclones and anticyclones these refinements have been omitted. We retain, then, simply

$$z - z_0 = 18400 (1 + .00367 \theta) \log \frac{B_0}{B} \dots \dots \dots \text{(II)}$$

where $\theta = \frac{T - T_0}{2} - 273^\circ$, the mean departure of the temperature of the air column from zero centigrade. This is to be compared with the formula,

$$-n \frac{k}{k-1} (\log T - \log T_0) = \log \frac{P_0}{P} = \log \frac{B_0}{B}, \dots \dots \text{(I)}$$

and it will be sufficient to show that

$$\frac{z - z_0}{18400 + 67.5 \theta} = -n \frac{k}{k-1} (\log T - \log T_0).$$

The examples are taken at random with sufficient range to test the formulæ severely. The observed gradient is found from $a = \frac{T - T_0}{z - z_0}$, per 1000 meters. The computation is given

in full as an example of the numerical quantities involved. While the agreement in the logarithms is not perfect, the differences I—II are small for so great ranges of temperature and height when translated into millimeters of mercury. If $B_0 = 760.00$ mm. in the fifth example, for the difference 0.00084 the value of B is 294.00 and 294.57, respectively. As my only purpose is to illustrate the numerical validity of the n formula, it will not be necessary to inquire further into the causes of the small differences between I and II.

COMPUTATION OF MEAN VALUES OF P_0 , ρ_0 , R_0 , FROM T_0 ON THE 1000-METER LEVELS.

In making an application of the formula (44),

$$\frac{P_0}{\rho_0} - \frac{P}{\rho} = \frac{1}{2} (q^2 - q_0^2) + g(z - z_0) = Q - Q_0 - C_p n (T - T_0) - C_p T \log T (n - n_0),$$

TABLE 15.—Comparison of the formulæ.

Formula I = $-\frac{k}{k-1} (\log T - \log T_0)$									
T	271.5	253.5	259.0	219.6	233.2	290.0	300.0	275.0	280.0
T_0	275.8	275.8	268.7	253.5	271.5	273.0	280.0	300.0	310.0
$T - T_0$	-4.3	-22.3	-9.7	-33.9	-38.3	+17.0	+20.0	-25.0	-30.0
Adiabatic.	-9.8695								
Observed.	-4.30	-4.46	-4.85	-6.78	-5.47	+4.25	+4.00	-5.00	-5.00
$-\log n$	-0.36082	-0.34496	-0.30855	-0.16306	-0.25630	0.36590	0.39223	-0.29532	-0.29532
$k/k-1$	0.53927								
$\log T$	2.43377	2.40398	2.41930	2.34163	2.36773	2.46240	2.47712	2.43933	2.44716
$\log T_0$	2.44059	2.44059	2.42927	2.40398	2.43377	2.43610	2.44716	2.47712	2.49136
$\log T - \log T_0$	-0.00682	-0.03661	-0.01997	-0.06235	-0.06604	0.02624	0.02996	-0.03779	-0.04420
$\log (\log T - \log T_0)$	-7.83378	-8.56360	-8.20330	-8.79484	-8.81981	8.41896	8.47654	-8.57738	-8.64542
$\log I$	8.73387	9.44783	9.05112	9.49717	9.61533	9.32413	9.40804	9.41197	9.48001
I	0.05418	0.28043	0.11249	0.31417	0.41246	0.21092	0.25588	0.25821	0.30200
Formula II = $\frac{z-z_0}{18400+67.5\theta}$									
$\frac{T+T_0}{2}$	273.65	264.65	263.85	236.55	252.35	281.5	290.0	287.5	295.0
θ	+0.65	-8.35	-9.15	-36.45	-20.65	+8.5	+17.0	+14.5	+22.0
$z-z_0$	1000	5000	2000	5000	7000	4000	5000	6000	6000
67.5θ	+44	-564	-618	-2460	-1394	+874	+1148	+979	+1485
K	18444	17836	17782	15940	17006	18974	19548	19379	19885
$K+67.5\theta$									
II	0.05422	0.28034	0.11247	0.31868	0.41162	0.21082	0.25578	0.25801	0.30173
I-II	-0.00004	+0.00009	+0.00002	+0.00049	+0.00084	+0.00010	+0.00010	+0.00020	+0.00027

to the earth's atmosphere, it is evident that we must first compute the values of P_0 , ρ_0 , R_0 , corresponding to T_0 as observed in the air while it is undisturbed by the local cyclonic and anticyclonic circulations. The observations of temperature were actually made in the midst of the prevailing local disturbances, but the average temperature of the air on the 1000-meter levels was found by taking the mean temperatures of the eight sectors, four in the high areas and four in the low areas, as in Tables 10 and 11. (See MONTHLY WEATHER REVIEW, February, 1906, Vol. XXXIV, p. 75.) Thus, Table 11 gives the adopted mean temperatures and gradients on the 1000-meter levels for American and European cyclones and anticyclones, and these data are used in the following computations.

The values of n are found on dividing 9.8695 by the gradient per 1000 meters, assuming that the gradient is a constant between the two levels. In a later section we shall compute, also, the values of n on the 1000-meter levels themselves, taking as the gradients those found on fig. 8 at the points indicated. We use the mean gradient between two levels in computing P_0 , ρ_0 , R_0 , and then the gradient at a given level to compute P , ρ , R , the abnormal values of pressure, density, and gas factor produced by the local cyclonic and anticyclonic disturbances. Having found n from level to level the following formulæ are applied in succession, including 41 as a check:

$$(38) \quad \log P = \log P_0 + n \frac{k}{k-1} (\log T - \log T_0).$$

$$(39) \quad \log \rho = \log \rho_0 + \frac{n}{k-1} (\log T - \log T_0).$$

$$(40) \quad \log R = \log R_0 + (n-1) (\log T - \log T_0).$$

$$(41) \quad \log \rho = \log \rho_0 + \frac{1}{k} (\log P - \log P_0).$$

A special point should be noted in connection with the symbols. P_0 is the pressure on one level, as the 1000-meter level, and then P is the pressure on another level, as the 2000-meter level, corresponding with T_0 and T , respectively. In this way a succession of values of P is found in the several strata of the undisturbed atmosphere, applying to the general circulation only. In studying the pressure variations in cyclones and anticyclones these P -pressures become P_0 in

formula (44), the P -values of that formula referring to disturbances within the local circulation on a given level. I have preferred to make this explanation rather than complicate the formulæ with additional symbols for all contingencies. In computing the tables following, the pressure $P_0 = 101323$, for $B = 760$ mm., was taken as the initial value. The initial value of the density on the sea-level plane was computed from,

$$\rho_0 = \frac{P_0}{R_0 T_0},$$

where $R_0 = 287.0334$, and T_0 is the value from Table 11, $T_0 = 275.8, 277.3, 290.2, 287.1$. While it is true that the atmosphere is seldom in the state represented by these tables, yet it fluctuates about these mean values, just as it does about the mean pressure, temperature, and density at sea level, and it is convenient to have reference values from which to conduct our discussions.

TABLE 16.—Computed values of the ratio n between successive 1000-meter levels.

Height in meters.	American.		European.	
	Winter.	Summer.	Winter.	Summer.
16000	3.037	2.820	3.037	3.589
14000	4.886	2.820	4.386	3.037
12000	2.078	1.716	2.078	1.645
10000	1.618	1.473	1.618	1.410
9000	1.390	1.410	1.410	1.316
8000	1.410	1.518	1.410	1.410
7000	1.451	1.673	1.410	1.518
6000	1.518	1.518	1.410	1.618
5000	1.794	1.410	1.673	1.702
4000	1.974	1.316	1.794	1.935
3000	2.100	1.653	2.014	2.295
2000	3.525	1.828	2.243	1.673
1000	2.295	1.828	2.467	1.645
000				

Since $n = \frac{-9.8695}{T - T_0}$, the variation of n in Table 16 is a function of ΔT . Where ΔT is large, n is small, and inversely. Hence, in the lower and the higher levels n is larger than in the middle levels, the change of temperature being slower below and above, for the reasons already given. In the middle levels $n = 1.5$ approximately, and it may become twice as great in higher or lower strata. Tables 17, 18, 19, and 20 contain the values of P_0 , ρ_0 , R_0 , T_0 on the several levels, and their logarithms which are useful in computations. Since $P_0 = g_0 \rho_m B_m$, we have

$$B_m = \frac{P_0}{g_0 \rho_m} = \frac{P_0}{9.806 \times 13595.8} = \frac{P_0}{133330} = \frac{P_0}{100} \times \frac{3}{4}, \text{ in millimeters.}$$

The pressure is higher in summer than in winter on the same level; the density is higher in winter than in summer; the gas factor is higher in summer than in winter; and the temperature is higher in summer than in winter, the difference diminishing in the upper levels. The gas factor is not a constant in any system except the adiabatic, where $n = 1$.

COLLECTION OF THE DATA SHOWING THE DISTRIBUTION OF THE DISTURBANCES ON THE 1000-METER LEVELS.

We will collect the data in a form suitable for the discussion of the distribution of the energy in cyclones and anticyclones, leaving the reader to make his own inferences by an examination of the tables in their relation to one another.

Computed mean values of the pressure, density, and gas factor from the temperature at several elevations.

TABLE 17.—EUROPEAN WINTER.

Height in meters.	P_0	$\log P_0$	ρ_0	$\log \rho_0$	R_0	$\log R_0$	T_0	$\log T_0$
16000.....	9547	3.97987	0.23733	9.37535	203.59	2.30874	197.6	2.29579
14000.....	13414	4.12756	0.30226	9.48038	217.45	2.33736	204.1	2.30984
12000.....	18679	4.27135	0.38250	9.58263	234.12	2.36943	208.6	2.31931
10000.....	25735	4.41052	0.48040	9.68160	245.63	2.39029	218.1	2.33366
9000.....	30097	4.47753	0.53610	9.72925	249.40	2.39688	224.6	2.35141
8000.....	34881	4.54259	0.59636	9.77551	252.55	2.40235	231.6	2.36474
7000.....	40335	4.60569	0.66127	9.82088	255.65	2.40765	238.6	2.37767
6000.....	46450	4.66699	0.73108	9.86397	258.70	2.41280	245.6	2.39023
5000.....	53276	4.72653	0.80595	9.90631	261.70	2.41780	252.6	2.40243
4000.....	60897	4.78461	0.88636	9.94761	265.80	2.42455	258.5	2.41246
3000.....	69403	4.84138	0.99536	9.98798	270.28	2.43181	264.0	2.42160
2000.....	78902	4.89709	1.06559	0.02759	275.37	2.43991	268.9	2.42959
1000.....	89502	4.95183	1.16551	0.06652	280.98	2.44367	273.3	2.43664
000.....	101323	5.00571	1.27300	0.10483	287.03	2.45793	277.3	2.44295

TABLE 18.—EUROPEAN SUMMER.

Height in meters.	P_0	$\log P_0$	ρ_0	$\log \rho_0$	R_0	$\log R_0$	T_0	$\log T_0$
16000.....	10187	4.00804	0.24003	9.38027	210.20	2.32262	201.9	2.30514
14000.....	14224	4.15302	0.30434	9.48336	225.34	2.35283	207.4	2.31681
12000.....	19674	4.29388	0.38329	9.58353	239.96	2.38013	213.9	2.33021
10000.....	26816	4.42888	0.47811	9.67953	248.55	2.39542	225.9	2.35392
9000.....	31156	4.49355	0.53152	9.72582	251.68	2.40085	232.9	2.36717
8000.....	35994	4.55623	0.58896	9.77009	254.21	2.40520	240.4	2.38093
7000.....	41408	4.61709	0.65069	9.81337	257.22	2.41031	247.4	2.39340
6000.....	47453	4.67627	0.71690	9.85546	260.70	2.41614	253.9	2.40466
5000.....	54201	4.73491	0.78802	9.89654	264.55	2.42241	260.0	2.41497
4000.....	61730	4.79050	0.86440	9.93671	268.68	2.42924	265.8	2.42455
3000.....	70118	4.84583	0.94638	9.97606	273.50	2.43696	270.9	2.43281
2000.....	79464	4.90017	1.03443	0.01470	279.14	2.44582	275.2	2.43965
1000.....	89846	4.95350	1.12882	0.05262	283.15	2.45202	281.1	2.44886
000.....	101323	5.00571	1.22956	0.08975	287.03	2.45793	287.1	2.45803

Computed mean values of the pressure, density, and gas factor from the temperature at several elevations.

TABLE 19.—AMERICAN WINTER.

Height in meters.	P_0	$\log P_0$	ρ_0	$\log \rho_0$	R_0	$\log R_0$	T_0	$\log T_0$
16000.....	9651	3.98456	0.24045	9.38103	201.59	2.30446	199.1	2.29907
14000.....	13527	4.13120	0.30571	9.48531	215.22	2.33289	205.6	2.31302
12000.....	18797	4.27408	0.38630	9.68691	231.60	2.36497	210.1	2.32243
10000.....	25833	4.41217	0.48430	9.68511	242.91	2.38544	219.6	2.34163
9000.....	30114	4.47876	0.54010	9.73247	246.61	2.39200	226.1	2.35430
8000.....	34945	4.54338	0.60037	9.77842	249.60	2.39724	233.2	2.36773
7000.....	40369	4.60605	0.66524	9.82298	252.64	2.40250	240.2	2.38057
6000.....	46450	4.66699	0.73505	9.86632	255.84	2.40797	247.0	2.39270
5000.....	53251	4.72633	0.81006	9.90852	259.31	2.41382	253.5	2.40399
4000.....	60836	4.78416	0.89052	9.94964	263.76	2.42121	259.0	2.41330
3000.....	69321	4.84087	0.97718	9.98997	268.71	2.42929	264.0	2.42160
2000.....	78317	4.89662	1.07059	0.02962	273.99	2.43773	268.7	2.42927
1000.....	89440	4.95153	1.17127	0.06866	281.25	2.44910	271.5	2.43377
000.....	101323	5.03571	1.27994	0.10719	287.03	2.45793	275.8	2.44059

TABLE 20.—AMERICAN SUMMER.

Height in meters.	P_0	$\log P_0$	ρ_0	$\log \rho_0$	R_0	$\log R_0$	T_0	$\log T_0$
16000.....	10232	4.00995	0.23822	9.37698	213.88	2.33017	200.8	2.30276
14000.....	14298	4.16529	0.30223	9.48033	227.65	2.35727	207.8	2.31765
12000.....	19754	4.29565	0.38031	9.58014	241.79	2.38344	214.8	2.33203
10000.....	26929	4.43022	0.47407	9.67584	250.99	2.39966	226.3	2.35468
9000.....	31252	4.49488	0.52701	9.72182	254.48	2.40566	233.0	2.36736
8000.....	36107	4.55759	0.58401	9.76642	257.59	2.41093	240.0	2.38021
7500.....	41554	4.61861	0.64537	9.80981	261.18	2.41694	246.5	2.39182
6000.....	47652	4.67808	0.71138	9.85210	265.37	2.42385	252.4	2.40209
5000.....	54449	4.73599	0.78213	9.89328	268.88	2.42956	258.9	2.41313
4000.....	62023	4.79255	0.85802	9.93350	271.84	2.43431	265.9	2.42472
3000.....	70402	4.84758	0.93892	9.97263	274.24	2.43813	273.4	2.43680
2000.....	79712	4.90152	1.02563	0.01099	278.16	2.44429	279.4	2.44623
1000.....	89968	4.95409	1.11782	0.04837	282.60	2.45117	284.8	2.45454
000.....	101323	5.00571	1.21642	0.08508	287.03	2.45793	290.2	2.46270

VALUES OF THE TEMPERATURES T , T_0 AND $T - T_0$.

The temperature data given in Tables 12 and 13 (see MONTHLY WEATHER REVIEW, February, 1906, Vol. XXXIV, pp. 76, 77) have been reproduced in Tables 21, 22, 23, and 24, in a form more convenient for carrying on the computations depending upon them. T is the temperature in the several sectors; T_0 is the mean temperature on the same level computed from the eight sectors of the correlative high and low areas; $T - T_0$ is the departure of the disturbed area from the assumed mean undisturbed atmosphere without any cyclonic and anticyclonic action, and is shown on figs. 5 and 6.

VALUES OF THE RATIOS n , n_0 AND $(n - n_0)$.

These are the ratios on the several 1000-meter levels, and they are used for computing the energy of the local disturbances on a given plane, rather than in reducing the elements from one plane to another, as was required for constructing Tables 17, 18, 19, and 20. The values of n on the several levels for each sector were scaled from fig. 8, for which purpose it was constructed, and from them the values of n in Tables 25, 26, 27, and 28 were computed. The values of n_0 , the mean ratio, were found by taking the means of the eight values of n in the sectors on the given plane. Tables 25, 26, 27, and 28 contain the data n , n_0 , and the differences $n - n_0$, the distribution of $n - n_0$ being given in fig. 14 on the several

levels. Since $n_0 = \frac{-9.8695}{T - T_0}$ is a certain average gradient, if n

Values of T , T_0 , $T - T_0$ derived from Tables 12, 13.

TABLE 21.—WINTER HIGH AREAS.

Height in meters.	T				T_0 Mean.	$T - T_0$			
	N.	E.	S.	W.		N.	E.	S.	W.
10000.....	$\circ C.$ -55.2	$\circ C.$ -56.2	$\circ C.$ -53.7	$\circ C.$ -52.2	$\circ C.$ -54.2	$\circ C.$ -1.0	$\circ C.$ -2.0	$\circ C.$ +0.6	$\circ C.$ +2.0
9000.....	-48.2	-60.2	-46.8	-45.3	-47.7	-0.5	-2.5	+0.9	+2.4
8000.....	-40.6	-43.6	-39.3	-37.6	-40.6	0.0	-3.0	+1.3	+3.0
7000.....	-32.9	-37.1	-32.1	-29.6	-33.6	+0.7	-3.5	+1.5	+4.0
6000.....	-25.3	-30.7	-25.5	-21.7	-26.7	+1.4	-4.0	+1.2	+5.0
5000.....	-18.0	-24.6	-20.0	-14.3	-20.0	+2.0	-4.6	0.0	+5.7
4000.....	-11.5	-18.6	-15.5	- 8.7	-14.8	+2.8	-4.3	-1.2	+5.6
3000.....	- 6.3	-13.0	-10.3	- 4.2	- 9.0	+2.7	-4.0	-1.3	+4.8
2000.....	- 2.2	- 8.0	- 6.9	- 0.2	- 4.2	+2.0	-3.8	-2.7	+4.0
1000.....	+ 0.9	- 3.7	- 3.1	+ 2.7	- 4.6	+1.5	-3.1	-2.5	+3.3
000.....	+ 4.2	+ 1.5	+ 0.1	+ 5.6	+ 3.6	+0.6	-2.1	-3.5	+2.0

TABLE 22.—WINTER LOW AREAS.

10000.....	-52.0	-51.2	-56.2	-57.0	-54.2	+2.2	+3.0	-2.0	-2.8
9000.....	-45.7	-44.7	-49.9	-50.7	-47.7	+2.0	+3.0	-2.2	-3.0
8000.....	-39.1	-37.9	-43.1	-43.9	-40.6	+1.5	+2.7	-2.5	-3.3
7000.....	-32.4	-31.1	-36.2	-37.1	-33.6	+1.2	+2.5	-2.6	-3.5
6000.....	-25.6	-24.5	-29.5	-30.5	-26.7	+1.1	+2.2	-2.8	-3.8
5000.....	-18.8	-18.0	-22.8	-24.0	-20.0	+1.2	+2.0	-2.8	-4.0
4000.....	-12.5	-12.5	-16.7	-17.8	-14.3	+1.8	+1.8	-2.4	-3.5
3000.....	- 6.8	- 7.6	-11.3	-12.4	- 9.0	+2.2	+1.4	-2.3	-3.4
2000.....	- 1.8	- 3.0	- 5.5	- 7.2	- 4.2	+2.4	+1.2	-1.3	-3.0
1000.....	+ 1.1	+ 0.7	+ 0.5	- 2.8	- 0.6	+1.7	+1.3	+1.1	-2.2
000.....	+ 4.5	+ 4.7	+ 6.5	+ 1.9	+ 3.6	+0.9	+1.1	+2.9	-1.7

TABLE 23.—SUMMER HIGH AREAS.

10000.....	-48.1	-48.4	-46.7	-45.9	-46.9	-1.2	-1.5	+0.2	+1.0
9000.....	-40.7	-42.1	-39.6	-38.6	-40.1	-0.6	-2.0	+0.5	+1.5
8000.....	-32.8	-35.3	-31.8	-31.0	-32.8	0.0	-2.5	+1.0	+1.8
7000.....	-25.4	-29.1	-24.6	-24.1	-26.1	+0.7	-3.0	+1.5	+2.0
6000.....	-18.5	-23.2	-18.7	-17.5	-19.9	+1.4	-3.3	+1.2	+2.4
5000.....	-11.7	-16.8	-13.7	-10.8	-13.6	+1.9	-3.2	-0.1	+2.8
4000.....	- 4.9	-10.4	- 7.7	- 5.0	- 7.2	+2.3	-3.2	-0.5	+2.2
3000.....	+ 1.0	- 3.8	- 1.3	+ 1.1	- 0.9	+1.9	-2.9	-0.4	+2.0
2000.....	+ 5.7	+ 2.1	+ 3.9	+ 5.9	+ 4.3	+1.4	-2.2	-0.4	+1.6
1000.....	+10.5	+ 8.1	+ 8.8	+11.0	+10.0	+0.5	-1.9	-1.2	+1.0
000.....	+15.9	+14.1	+13.7	+16.1	+15.7	+0.2	-1.6	-2.0	+0.4

TABLE 24.—SUMMER LOW AREAS.

10000.....	-42.1	-45.7	-48.9	-49.4	-46.9	+4.8	+1.2	-2.0	-2.5
9000.....	-35.5	-38.9	-42.4	-42.9	-40.1	+4.6	+1.2	-2.3	-2.8
8000.....	-28.4	-31.8	-35.5	-35.8	-32.8	+4.4	+1.0	-2.7	-3.0
7000.....	-21.9	-24.9	-29.1	-29.3	-26.1	+4.2	+1.2	-3.0	-3.2
6000.....	-15.9	-18.5	-22.9	-23.4	-19.9	+4.0	+1.4	-3.0	-3.5
5000.....	-10.3	-12.2	-16.3	-16.6	-13.6	+3.3	+1.4	-2.7	-3.0
4000.....	- 4.4	- 5.9	- 9.2	- 9.8	- 7.2	+2.8	+1.3	-2.0	-2.6
3000.....	+ 1.5	- 0.6	- 1.9	- 3.5	- 0.9	+2.4	+0.3	-1.0	-2.6
2000.....	+ 6.2	+ 4.5	+ 4.3	+ 1.9	+ 4.3	+1.9	+0.2	0.0	-2.4
1000.....	+11.2	+10.4	+11.0	+ 7.8	+10.0	+1.2	+0.4	+1.0	-2.2
000.....	+16.5	+16.5	+18.5	+14.2	+15.7	+0.8	+0.8	+2.8	-1.5

Distribution of the values of n , n_0 , $n - n_0$.

TABLE 25.—WINTER HIGH AREAS.

Height in meters.	n				n_0 Mean.	$n - n_0$			
	N.	E.	S.	W.		N.	E.	S.	W.
10000.....	1.507	1.778	1.574	1.535	1.600	-.093	+.178	-.026	-.065
9000.....	1.339	1.537	1.367	1.319	1.444	-.105	+.093	-.077	-.125
8000.....	1.282	1.495	1.348	1.246	1.396	-.114	+.090	-.048	-.150
7000.....	1.283	1.523	1.404	1.234	1.403	-.120	+.120	+.001	-.169
6000.....	1.323	1.567	1.613	1.272	1.459	-.136	+.108	+.154	-.187
5000.....	1.420	1.623	2.100	1.430	1.602	-.182	+.021	+.498	-.172
4000.....	1.747	1.725	2.317	1.974	1.819	-.072	-.094	+.498	+.155
3000.....	2.274	1.894	2.804	2.443	2.105	+.169	-.211	+.699	+.338
2000.....	2.920	2.285	2.937	3.056	2.554	+.866	-.269	+.383	+.502
1000.....	3.184	2.179	2.903	3.439	2.661	+.623	-.482	+.242	+.778
000.....	2.991	1.769	2.467	3.290	2.403	+.588	-.634	+.064	+.887

TABLE 26.—WINTER LOW AREAS.

10000.....	1.587	1.547	1.613	1.659	1.600	-.013	-.053	+.018	+.059
9000.....	1.628	1.458	1.600	1.500	1.444	+.034	+.014	+.056	+.056
8000.....	1.471	1.447	1.437	1.443	1.396	+.075	+.051	+.041	+.047
7000.....	1.430	1.471	1.430	1.447	1.403	+.027	+.068	+.027	+.044
6000.....	1.426	1.518	1.458	1.498	1.459	-.033	+.059	-.001	+.039
5000.....	1.489	1.629	1.664	1.562	1.602	-.113	+.027	-.038	-.040
4000.....	1.629	1.766	1.707	1.690	1.819	-.190	-.053	-.112	-.129
3000.....	1.848	1.974	1.725	1.876	2.105	-.257	-.131	-.380	-.229
2000.....	2.903	2.618	1.667	2.150	2.554	+.847	-.036	-.887	-.404
1000.....	3.123	2.611	1.637	2.213	2.661	+.462	-.050	-1.024	-.448
000.....	2.632	2.367	1.645	2.065	2.403	+.229	-.036	-.758	-.338

TABLE 27.—SUMMER HIGH AREAS.

10000.....	1.371	1.623	1.430	1.386	1.503	-.132	+.120	-.073	-.117
9000.....	1.233	1.482	1.307	1.328	1.382	-.149	+.100	-.075	-.054
8000.....	1.265	1.491	1.309	1.360	1.396	-.131	+.095	-.037	-.036
7000.....	1.392	1.662	1.537	1.469	1.539	-.147	+.123	-.002	-.070
6000.....	1.443	1.667	1.873	1.673	1.628	-.185	+.039	+.245	+.045
5000.....	1.447	1.589	1.905	1.725	1.607	-.160	-.018	+.298	+.118
4000.....	1.542	1.505	1.577	1.631	1.563	-.021	-.058	+.014	+.068
3000.....	1.947	1.569	1.645	1.818	1.728	+.219	-.159	-.083	+.090
2000.....	2.113	1.631	1.909	1.974	1.869	+.244	-.188	+.040	+.105
1000.....	1.905	1.637	1.974	1.909	1.753	+.152	-.116	+.221	+.156
000.....	1.785	1.645	2.014	1.902	1.688	+.097	-.043	+.326	+.214

TABLE 28.—SUMMER LOW AREAS.

10000.....	1.607	1.500	1.572	1.535	1.503	+.104	-.003	+.069	+.032
9000.....	1.443	1.394	1.445	1.422	1.382	+.061	+.012	+.063	+.040
8000.....	1.439	1.398	1.478	1.426	1.396	+.043	+.002	+.082	+.030
7000.....	1.559	1.516	1.597	1.582	1.539	+.020	-.023	+.058	+.043
6000.....	1.725	1.574	1.564	1.507	1.628	+.097	-.054	-.064	-.121
5000.....	1.744	1.569	1.430	1.447	1.607	+.137	-.038	-.177	-.160
4000.....	1.684	1.719	1.352	1.491	1.563	+.121	+.156	-.211	-.072
3000.....	1.769	1.943	1.386	1.744	1.728	+.041	+.215	-.342	+.016
2000.....	2.109	1.855	1.557	1.750	1.869	+.440	-.014	-.312	-.119
1000.....	1.974	1.673	1.384	1.572	1.753	+.221	-.080	-.369	-.181
000.....	1.781	1.597	1.279	1.502	1.688	+.093	-.091	-.409	-.186

Distribution of the heights $z-z_0$. $(z-z_0)g=-C_p n_0(T-T_0)$

Distribution of the velocities $q, q_0, \frac{1}{2}(q^2-q_0^2)$.

TABLE 29.—WINTER HIGH AREAS.

TABLE 33.—WINTER HIGH AREAS.

Height in meters.	$-C_p n_0(T-T_0)$				g	$z-z_0$				Height in meters.	q				q_0 Mean.	$\frac{1}{2}(q^2-q_0^2)$			
	N.	E.	S.	W.		N.	E.	S.	W.		N.	E.	S.	W.		N.	E.	S.	W.
10000.....	1590	3180	— 795	—3180	9.786	+162	+325	— 81	— 325	10000.....	44	39	31	34	37.6	+261	+54	—227	—127
9000.....	717	3587	—1291	—3443	9.789	+ 73	+366	—132	— 352	9000.....	43	38	28	33	36.5	+259	+56	—274	—122
8000.....	0	4161	—1803	—4161	9.791	0	+425	—184	— 425	8000.....	41	37	26	32	35.0	+226	+72	—275	—101
7000.....	— 976	4879	—2091	—5576	9.793	—100	+498	—213	— 569	7000.....	39	35	23	30	32.9	+220	+72	—277	— 91
6000.....	—2030	5799	—1739	—7248	9.795	—207	+592	—178	— 740	6000.....	37	32	20	27	29.9	+238	+65	—247	— 83
5000.....	—3183	7322	0	—9075	9.796	—325	+748	0	— 926	5000.....	34	30	17	24	26.9	+216	+88	—218	— 74
4000.....	—5060	7772	2169	—10121	9.798	—516	+793	+221	—1033	4000.....	32	28	16	23	25.3	+192	+72	—192	— 56
3000.....	—5647	8366	2719	—10039	9.800	—576	+854	+278	—1024	3000.....	30	26	14	21	23.6	+172	+60	—181	— 58
2000.....	—5075	9543	6851	—10150	9.802	—518	+984	+699	—1036	2000.....	25	20	12	18	19.5	+123	+10	—118	— 28
1000.....	—3966	8196	6610	—8725	9.804	—405	+836	+674	— 890	1000.....	19	15	10	14	14.8	+ 71	+ 3	— 60	— 12
000.....	—1433	5014	8356	—4775	9.806	—146	+511	+853	— 487	000.....	9	9	6	7	8.0	+ 8	+ 8	— 14	— 8

TABLE 30.—WINTER LOW AREAS.

TABLE 34.—WINTER LOW AREAS.

10000.....	—3497	—4769	3180	4451	9.786	—357	—487	+325	+454	10000.....	31	35	45	42	37.6	—227	— 95	+306	+175
9000.....	—2869	—4304	3156	4304	9.789	—280	—440	+322	+440	9000.....	30	33	45	42	36.5	—216	—122	+347	+216
8000.....	—2081	—3745	3468	4577	9.791	—213	—382	+354	+467	8000.....	28	31	44	41	35.0	—221	—132	+356	+228
7000.....	—1673	—3485	3624	4879	9.793	—171	—356	+370	+498	7000.....	26	29	42	39	32.9	—203	—121	+342	+220
6000.....	—1595	—3189	4059	5509	9.795	—163	—326	+414	+563	6000.....	23	27	38	35	29.9	—183	— 83	+275	+166
5000.....	—1910	—3183	4457	6367	9.796	—195	—325	+455	+650	5000.....	21	25	34	30	26.9	—142	— 50	+216	+ 88
4000.....	—3253	—3253	4338	6326	9.798	—332	—332	+443	+646	4000.....	19	24	32	28	25.3	—140	— 32	+192	+ 72
3000.....	—4601	—2928	4810	7111	9.800	—591	—299	+491	+726	3000.....	18	23	30	27	23.6	—117	— 14	+172	+ 86
2000.....	—6090	—3045	3299	7613	9.802	—621	—311	+337	+777	2000.....	15	18	26	22	19.5	— 78	— 28	+148	+ 52
1000.....	—4495	—3437	—2908	5817	9.804	—459	—351	—297	+593	1000.....	12	13	20	16	14.8	— 38	— 25	+ 96	+ 19
000.....	—2149	—2626	—6924	4059	9.806	—219	—268	—706	+414	000.....	6	7	10	10	8.0	— 16	— 8	+ 18	+ 18

TABLE 31.—SUMMER HIGH AREAS.

TABLE 35.—SUMMER HIGH AREAS.

10000.....	1794	2240	— 299	—1492	9.786	+183	+229	— 31	—153	10000.....	37	33	26	29	33.5	+124	—12	—223	—141
9000.....	824	2746	— 687	—2060	9.789	+ 84	+280	— 70	—211	9000.....	35	32	24	28	32.5	+120	—16	—240	—136
8000.....	0	3468	—1387	—2497	9.791	0	+354	—142	—255	8000.....	35	31	22	27	31.1	+129	— 3	—242	—119
7000.....	—1071	4587	—2294	—3058	9.793	—109	+468	—234	—312	7000.....	33	29	19	25	29.0	+124	0	—240	—108
6000.....	—2265	5338	—1941	—3882	9.795	—231	+545	—198	—397	6000.....	31	27	17	23	26.5	+130	+14	—207	— 87
5000.....	—3034	5109	160	—4471	9.796	—312	+522	+ 16	—457	5000.....	29	25	14	20	23.9	+135	+27	—188	— 86
4000.....	—3572	4970	777	—3416	9.798	—364	+507	+ 79	—349	4000.....	27	24	13	19	22.4	+114	+37	—166	— 71
3000.....	—3262	4979	687	—3434	9.800	—333	+508	+ 70	—350	3000.....	25	22	12	18	21.0	+ 92	+22	—124	— 59
2000.....	—2600	4085	743	—2971	9.802	—265	+417	+ 76	—308	2000.....	21	17	10	15	17.3	+ 71	— 5	—100	— 37
1000.....	— 871	3309	2090	—1742	9.804	— 88	+338	+213	—178	1000.....	16	12	8	12	13.1	+ 42	—14	— 54	— 14
000.....	— 669	2683	3354	— 671	9.806	— 68	+274	+342	— 68	000.....	8	7	5	6	7.3	+ 6	— 2	— 14	— 9

TABLE 32.—SUMMER LOW AREAS.

TABLE 36.—SUMMER LOW AREAS.

10000.....	—7168	—1792	2987	3733	9.786	—732	—183	+305	+381	10000.....	29	33	42	39	33.5	—141	—17	+321	+200
9000.....	—6316	—1648	3158	3845	9.789	—645	—168	+322	+393	9000.....	28	31	42	39	32.5	—136	—47	+354	+233
8000.....	—6103	—1387	3745	4161	9.791	—623	—142	+382	+425	8000.....	26	29	41	38	31.1	—146	—63	+357	+239
7000.....	—6422	—1835	4587	4893	9.793	—656	—169	+468	+500	7000.....	24	27	39	36	29.0	—183	—56	+340	+228
6000.....	—6470	—2265	4864	5661	9.795	—661	—231	+496	+577	6000.....	21	25	35	33	26.5	—231	—39	+262	+194
5000.....	—5269	—2236	4311	4790	9.796	—538	—228	+440	+489	5000.....	20	23	32	28	23.9	— 86	—21	+227	+107
4000.....	—4348	—2019	3106	4038	9.798	—444	—207	+317	+412	4000.....	18	22	30	26	22.4	— 89	— 9	+199	+ 87
3000.....	—4120	— 515	1717	4464	9.800	—420	— 53	+175	+455	3000.....	17	21	28	25	21.0	— 76	0	+172	+ 92
2000.....	—3528	— 371	0	4457	9.802	—360	— 38	0	+454	2000.....	14	17	24	20	17.3	— 52	— 5	+139	+ 51
1000.....	—2090	— 697	—1742	3832	9.804	—213	— 71	—178	+390	1000.....	11	12	19	15	13.1	— 26	—14	+ 95	+ 27
000.....	—1342	—1342	—4696	2481	9.806	—137	—137	—479	+253	000.....	6	7	9	9	7.3	— 9	— 2	+ 14	+ 14

Distribution of the heat, $Q - Q_0$, and the pressure, $B - B_0$.

TABLE 37.—WINTER HIGH AREAS.

Height in meters.	$Q - Q_0$				$B - B_0$			
	N.	E.	S.	W.	N.	E.	S.	W.
10000	-11.2	+21.7	-3.3	-8.0	-10.1	-18.2	+5.5	+19.5
9000	-13.2	+11.7	-9.8	-15.8	-5.7	-22.8	+9.6	+23.4
8000	-14.9	+13.0	-6.4	-19.7	-1.1	-30.1	+14.7	+32.1
7000	-16.2	+16.3	+0.1	-22.9	+7.1	-39.5	+19.2	+49.2
6000	-19.0	+15.1	+21.5	-26.2	+17.5	-51.7	+17.8	+71.7
5000	-26.3	+3.1	+71.9	-24.9	+34.5	-73.9	+1.3	+104.8
4000	-10.7	-13.9	+73.9	+23.0	+60.1	-75.4	-23.6	+128.3
3000	+25.7	-32.1	+106.0	+51.4	+69.1	-94.8	-30.6	+129.8
2000	+56.8	-41.7	+59.0	+77.9	+83.2	-140.0	-102.1	+176.9
1000	+82.5	-76.0	+38.1	+122.5	+58.1	-112.7	-91.4	+137.2
000	+94.4	-101.6	+10.3	+142.5	+15.9	-57.1	-92.9	+55.5

TABLE 38.—WINTER LOW AREAS.

10000	-1.6	-6.5	+1.7	+7.2	+21.8	+12.0	-19.1	-25.6
9000	+10.5	-1.8	+7.1	+7.1	+19.9	+29.3	-21.3	-26.3
8000	+9.8	+6.6	+5.5	+6.2	+15.8	+29.0	-26.4	-33.4
7000	+3.6	+9.2	+3.8	+6.0	+15.2	+30.6	-31.1	-40.2
6000	-4.7	+8.2	0.0	+5.5	+16.0	+30.9	-37.9	-49.8
5000	-16.4	+3.9	-5.5	-5.8	+21.6	+35.1	-46.9	-57.3
4000	-28.2	-7.9	-16.6	-9.1	+39.9	+39.2	-50.6	-70.7
3000	-39.1	-19.9	-57.7	-34.8	+57.9	+35.9	-56.9	-81.6
2000	+53.9	-5.6	-137.5	-62.7	+102.7	+53.9	-51.9	-113.3
1000	+72.8	-7.9	-161.2	-70.6	+67.2	+51.0	+41.9	-81.4
000	+36.8	-5.8	-121.5	-54.3	+25.2	+30.4	+80.7	-46.0

TABLE 39.—SUMMER HIGH AREAS.

10000	-16.7	+15.2	-9.3	-14.8	-10.8	-12.2	+2.4	+9.4
9000	-19.5	+13.1	-9.9	-7.1	-6.0	-17.9	+6.3	+14.5
8000	-17.8	+12.9	-11.8	-5.0	-07.6	-25.7	+11.8	+20.0
7000	-20.6	+17.3	-0.3	-9.9	+8.5	-37.3	+20.8	+26.9
6000	-26.7	+5.6	+35.4	+6.5	+18.9	-43.8	+17.9	+34.1
5000	-23.8	-2.8	+44.3	+17.6	+26.6	-43.6	-0.3	+40.9
4000	-31.9	-9.0	+2.1	+10.4	+33.1	-45.0	-6.6	+41.3
3000	+34.5	-25.0	-13.1	+14.2	+34.6	-51.2	-6.4	+37.5
2000	+39.2	-30.4	+6.7	+16.9	+32.2	-49.3	-8.4	+37.8
1000	+25.1	-19.2	+36.5	+25.7	+12.7	-48.2	-30.3	+26.5
000	+16.4	-7.2	+55.0	+36.2	+4.5	-36.1	-44.9	+9.2

TABLE 40.—SUMMER LOW AREAS.

10000	+13.1	-0.4	+8.8	+4.1	+45.1	+11.2	-18.2	-21.9
9000	+8.0	+1.6	+8.3	+5.3	+45.2	+11.3	-20.9	-25.8
8000	+5.8	+0.3	+11.2	+4.2	+64.9	+11.1	-29.3	-31.8
7000	+2.8	-3.3	+8.2	+6.1	+57.7	+16.0	-38.8	-41.1
6000	+14.0	-7.8	-9.2	-17.5	+58.2	+19.8	-41.2	-47.3
5000	+20.4	-5.7	-26.3	-23.8	+48.4	+20.0	-38.1	-40.4
4000	+18.5	+23.9	-32.3	-11.0	+43.2	+34.1	-29.6	-37.1
3000	+6.5	+33.9	-53.8	+2.8	+45.1	+5.5	-19.2	-46.6
2000	+38.6	-2.3	-50.2	-19.2	+45.1	+6.0	-1.1	-54.1
1000	+36.5	-13.2	-60.9	-29.9	+31.9	+10.6	+25.5	-56.0
000	+15.7	-15.4	-69.1	-31.4	+19.1	+18.4	+64.9	-34.1

is larger than n_0 and $n - n_0$ is positive, it follows that the temperature gradient is smaller for n than for n_0 , and if n is smaller than n_0 the temperature gradient is larger in the disturbed region than in the normal undisturbed stratum. An examination of the tables and the fig. 14 shows that while the distribution of $n - n_0$ is similar to that of $T - T_0$ as to the sectors, yet there is a *distinct reversal between the lower and the higher strata*, occurring near the 4000-meter level. The positive (+) values of $n - n_0$ in the lower strata, showing a decrease of gradient of temperature or an inflow of heat on the western side of the high areas, become negative (-) values in the upper strata, where they indicate a more rapid loss of the temperature. The reverse conditions hold on the western half of the low areas from the surface to 10,000 meters. Hence an inflow of warm air in the lower strata, that is the southerly current, diminishes the normal gradient of temperature and produces $n - n_0$ which is a (+) quantity; similarly, an inflow of cold air in the higher strata, that is of cold air from the north, diminishes the normal temperature gradient and produces a value of $n - n_0$ which is also positive. These conditions are therefore in harmony with the observed temperature distribution in cyclones and anticyclones. The reversal between the lower and higher strata of the same sectors indicates the thermodynamic effect of the air masses striving to return to an equilibrium by reversing the gradients above and below the level of 4000 meters.

VALUES OF THE TERMS $-C_p n (T - T_0)$ AND $(z - z_0)$.

Having determined the distribution of $(T - T_0)$ and $(n - n_0)$ in the several strata, since all the terms of the equation, except the velocity, depend upon them, we proceed to compute these terms for the sake of ultimately finding the heat variations $Q - Q_0$, and the barometric pressure variations $B - B_0$. The first condition to be found is $g(z - z_0) = -C_p n_0 (T - T_0)$ which represents the available potential energy of the air mass at a given level, to be expended in producing the kinetic energy found in cyclonic and anticyclonic circulations. Tables 29, 30, 31, 32, and fig. 15 contain the several terms. $-C_p n_0 (T - T_0)$ is the potential energy, due to the fact that the mass of temperature T on the level where T_0 is the normal temperature can be reduced to equilibrium by rising or falling through a given height under the gravity acceleration g , where $g(z - z_0)$ is the work to be expended in falling or rising through the height $(z - z_0)$. The value of g is given on each 1000-meter level. The height $(z - z_0)$ is computed from the formula, and by fig. 15 one can see that it has the same distribution as $(T - T_0)$, upon which it depends. Thus, the positive (+) sign for $(z - z_0)$ indicates that the air mass is too cold for its level and that it can fall through a given height before reaching the normal temperature of its stratum; similarly, the negative (-) sign for $(z - z_0)$ shows that the air mass is too warm for its level and can rise through a given height to reach equilibrium. The cold sectors have the (+) sign, and the warm sectors the (-) sign, and hence the entire cold column is able to fall and the entire warm column can rise through $(z - z_0)$ meters under the influence of gravity. This is the primary source of the energy of motion in cyclones and anticyclones, this potential energy being converted into pressure differences and motions.

An inspection of the tables and the fig. 15 shows that the maximum potential energy is on the east and west sectors, at the boundary of the high and low pressure areas. Hence, there is a gradient of potential within the cold areas from the north toward the southeast or south, and within the warm areas from the south toward the northeast and north at all the levels except that next the surface. The difference between the lowest level and those above it must represent a reaction from the ground, and an accumulation of the dynamic effects from the other forces yet to be considered, such as those from

the horizontal components, the deflecting force, the friction, and the other dynamic effects. Primarily, we must recognize that we deal here with a couple, one branch from the north on the west of the cyclone, and the other from the south on the east of the cyclone. The complex interactions which occur in consequence of these dispositions of warm and cold air masses form an important and difficult subject of study in hydrodynamics, which must be considered in a later paper.

DISTRIBUTION OF THE VELOCITIES $q, q_0, \frac{1}{2}(q^2 - q_0^2)$.

Referring to the figs. 6 and 7, MONTHLY WEATHER REVIEW, March, 1902, for the vectors in cyclones and anticyclones, which were derived from the cloud observations made by the Weather Bureau in 1896-1897, I have transferred the values for the several gradients to Tables 33, 34, 35, and 36 in the first section q ; the mean value of the q -velocity in meters per second in the eight sectors of the corresponding high and low areas gives the mean value q_0 ; from these are computed $\frac{1}{2}(q^2 - q_0^2)$ which are plotted on fig. 16. The distribution shows that there is a prevailing northwest cold current between the high and low areas, and a southeast warm current between the low and high areas. The maximum values of $\frac{1}{2}(q^2 - q_0^2)$ occur in the strata that are elevated 6000-8000 meters above the surface, and the values are somewhat greater in the cold than in the warm areas. Attention is called to the small number of units of force which are actually expended in changing the prevailing velocities of the eastward drift, in comparison with the energy available in the potential of the preceding terms.

THE DISTRIBUTION OF THE HEAT, $Q - Q_0$.

The heat term is computed from the formula,

$$Q - Q_0 = \frac{1}{2}(q^2 + q_0^2) + C_p T_0 \log T_0 (n - n_0),$$

using the values given in the preceding tables, first in mechanical units, which are then converted into calories by the divisor 4185.57. The results are tabulated in Tables 37, 38, 39, and 40, and the distribution is shown in fig. 17, which is quite similar to that for $(n - n_0)$ of fig. 14, since the term $Q - Q_0$ depends chiefly upon $(n - n_0)$. There is, therefore, the same reversal noted as in the previous case, the positive sign (+) denoting a potential energy to be turned into heat and the negative sign (-) an amount of energy to lose by cooling. One can not but be impressed with the very large amount of heat energy here available, in comparison with which the kinetic energy of motion is insignificant.

This may be the proper place to speak of the *efficiency of the atmosphere as a thermal engine*. By analogy, the warm masses are in the boiler and the cold masses are in the condenser. The percentage of heat actually turned into kinetic energy seems to be very small and the efficiency is not large. This probably comes from the general circumstance that air masses can lose their thermal contents only by action on their edges or surfaces, the interior of each mass holding its individuality for a long time under the forces which tend to destroy it by working slowly into its interior. The efficiency of the warm and cold masses under atmospheric conditions is evidently a subject which will demand very careful consideration before it can be fully analyzed and expressed as a mathematical function.

DISTRIBUTION OF THE PRESSURES, $B - B_0$.

We approach this term through the formula

$$P = \rho \left(\frac{P_0}{\rho_0} - \left[\frac{1}{2}(q^2 - q_0^2) + g(z - z_0) \right] \right).$$

The term $\frac{P_0}{\rho_0}$ is computed from the data of Tables 17 and 18, because we have no way to determine the mean pressure P_0

and the mean density P_0 of the undisturbed stratum, except by assuming the normal conditions there found, and supposing that the local variations in pressure should be measured from them. It was for this purpose that these tables were computed, in order that they might ultimately be used in studying

the local cyclonic and anticyclonic variations. As $\frac{P_0}{\rho_0}$ can be

readily computed, its values will not be introduced into this paper. Furthermore, it is necessary to compute the local density, ρ , by the formula

$$\log \rho = \log \rho_0 + \frac{n}{k-1} (\log T - \log T_0)$$

the n being taken as the mean n of Table 16. These values are also omitted because of the magnitude of the tables that would be required to reproduce them in this place. We finally obtain the values $P - P_0$, the variation of the pressure in mechanical units, which may be converted into millimeters by the formula,

$$B = \frac{P}{100} \times \frac{3}{4}.$$

The resulting values, $B - B_0$, are given in the second section of Tables 37, 38, 39, and 40 and they are plotted on fig. 18. The distribution is again such as has been made familiar in the preceding figures of these papers.

These pressure differences are given in millimeters, and they represent a potential energy which can be converted into cyclonic and anticyclonic motions and pressures. The fact that the observed pressures are not so great as those here given shows that the efficiency of the kinetic structure is not so great as the potential energy would indicate. Not all the available energy goes into storms, a portion being carried along in the circulating structures without transformation and a part being frittered away in internal work agitations. We have, however, shown that there seems to be an abundant supply of energy in warm and cold masses of different temperatures in the neighborhood of each other, sufficient to account for all the phenomena observed by meteorologists. It has been proven that the primary distribution is asymmetrical in respect to the centers of the low and high pressure areas. It is now one of the difficult problems to show, mathematically, how the action of these cold and warm masses, arranged as couples between the dynamic centers, is transformed from the thermodynamic structures here indicated into the hydrodynamic structures actually existing in the atmosphere. It should be remembered that the closed isobars of the lower strata, practically symmetrical about the high and low centers, are quickly modified above the surface into loops wherein the distribution of the pressure is entirely different from that at the surface as shown on the sea-level weather charts. The construction of daily and monthly isobars on the 3500-foot plane and the 10,000-foot plane for the United States during the year 1903 made this change of the structure of the isobars familiar to me. It is next in order to discuss the equations in the horizontal plane, namely,

$$q \frac{dq}{d\sigma} = \frac{dQ}{d\sigma} - c_p n_0 \frac{dT}{d\sigma} - c_p T_0 \log T_0 \frac{dn}{d\sigma}$$

where σ is a line in the plane dx, dy ; together with the deflecting forces represented by the terms,

$$- \cos \theta (2\omega + \nu) v \partial x + \cos \theta (2\omega + \nu) u \partial y,$$

which are equal to zero, so far as the circulation is concerned, since $v \partial x = u \partial y$, though they have a decided effect upon the position of the resulting isobars; and, finally, the unknown terms representing the secondary or vortical motions induced by the dynamic motions in the sensitive hydrodynamic medium of the atmosphere.

FIG. 14.—Distribution of the values of $n - n_0$ in the high and low areas.

FIG. 15.—Distribution of $z - z_0 = -\frac{C_p n_0}{g} (T - T_0)$.

Fig. 16.—Distribution of the velocity term $\frac{1}{2}(q^2 - q_0^2)$.

FIG. 17.—Distribution of the heat, $Q - Q_0$.

FIG. 18.—Distribution of the pressure, $B - B_0$.

STUDIES ON THE THERMODYNAMICS OF THE ATMOSPHERE.

By Prof. FRANK H. BIGELOW.

V.—THE HORIZONTAL CONVECTION IN CYCLONES AND ANTICYCLONES.¹

SOME OF THE DIFFICULTIES IN THIS PROBLEM.

If one wishes to follow the exact process occurring in the natural circulation of the atmosphere, then the next step in the orderly development of the analysis of the problem of the structure of cyclones and anticyclones is exceedingly difficult, and some time must elapse before meteorologists will be able to complete the solution in a rigorous manner. This may be explained by resuming our study of the interchange of energy in the nonadiabatic circulation between high and low areas.² Equations (44) and (52), so far as they relate to the circulation in a horizontal plane xy , in the integrated form give the following:

$$C_p n_0 (T - T_0) + C_p T_0 \log T_0 (n - n_0) = (Q - Q_0) - \frac{1}{2} (q^2 - q_0^2).$$

Since there is to be an interchange of energy between the cold area, whose center will be marked C , and the warm area whose center is W , the following notation will be employed:

$$\left. \begin{array}{l} n_0, \text{ the gradient ratio}^3 \\ T_0, \text{ the temperature} \\ q_0, \text{ the vector velocity} \\ Q_0, \text{ the heat energy} \end{array} \right\} \text{ in the cold area, } C.$$

$$\left. \begin{array}{l} n, \text{ the gradient ratio}^3 \\ T, \text{ the temperature} \\ q, \text{ the vector velocity} \\ Q, \text{ the heat energy} \end{array} \right\} \text{ in the warm area, } W.$$

The C and W areas lie between the centers of high and low pressure, marked H and L , respectively, in the order from west to east, as follows:

H (high); C (cold); L (low); W (warm);

as illustrated in the diagrams of papers No. I, II, III, and IV of this series. (MONTHLY WEATHER REVIEW, 1906, January, February, March, and June, respectively.)

(A) One problem is to show the relations between the thermodynamic centers C and W , and the hydrodynamic centers H and L in the moving atmosphere. It will not be proper to make model circulations by erecting chambers around given masses, and then removing certain internal partitions. This process really evades the entire problem to be solved, and substitutes some ideal or experimental system in place of that occurring in the atmosphere.

(B) Another problem is concerned with the gradient factors (n_0 and n) and the temperatures (T_0 and T), and may be stated in the following form. Since the gradients of temperature are changing from point to point in the vertical and in the horizontal directions in a very complex fashion, it seems impracticable to assign temperature functions in advance of the actual observations, and therefore analytic formulas of sufficient flexibility to express the entire existing conditions are impossible. If a simple function of the temperature is adopted, it is certain that this function will not be applicable to the cyclonic structure taken as a whole, and hence it is very hard to derive the pressures from the temperatures by the simple quasi-adiabatic formulas.

(C) Furthermore, the most troublesome problem of all, in the present state of meteorology, is to show what is the relation between the velocity terms (q_0 and q) and the heat terms (Q_0 and Q). The cyclonic circulation constitutes an effort to bring back to equilibrium the energy-difference represented in the cold and warm areas, and this is done by setting up an

extensive series of internal vortices, graduated in size from the large storm areas, down thru tornadoes or secondaries to the minute whirls that are not accessible to any instrumental records. In this interchange of heat between the warm and cold masses, a portion of the energy is absorbed in maintaining the velocity of the masses of air, a second portion goes into radiation, and a third part into equalizing the temperatures. The velocity of the wind in a cyclone does not measure the true velocities (q_0 and q), since the latter include the total internal circulation as well as the flow of the main stream; but there seems to be no way to separate these parts from one another. In a word the total energy is given by the terms

$$C_p n_0 (T - T_0) + C_p T_0 \log T_0 (n - n_0),$$

but I can as yet discover no method of distributing the respective portions of this total among the equivalent terms,

$$(Q - Q_0) - \frac{1}{2} (q^2 - q_0^2) + \text{radiation}.$$

Until all these difficulties have been overcome it will be possible to make only tentative and incomplete discussions of the great problem involved in analytic meteorology.

(D) Finally, the general question as to the reason why the observed gradients of temperature differ from the adiabatic gradient is closely bound up with the distribution of the available energy between the q and Q terms. If a mass of air is moved from one level to another, as from 5000 meters to 4000 meters, in an adiabatic atmosphere, the pressure and the temperature change according to the adiabatic law; in a non-adiabatic atmosphere, the change of temperature does not correspond with the pressure, but a divergence exists depending on the proportion represented by the difference of the ratios $n - n_0$. If in a nonadiabatic atmosphere there is vertical displacement of an air mass, the interchange of energy is partly as heat and partly as velocity, and at the moment a mass moving adiabatically in the midst of a nonadiabatic mass arrives at such a displacement, $z - z_0$, as to be appreciable in respect to $n - n_0$, there is set up a small local interchange of energy between these masses in the form of a minor gyration of some sort. There is thus a continual tendency to balance these two expenditures of energy, the one against the other, in the most economical way, and the resultant temperature and circulation represents the outcome of this physical process. (See fig. 19.)

If instead of one rising current of warm air, $A C$, which becomes overcooled, and one current of cold air, $E W$, which becomes overheated by adiabatic expansion and contraction with the change of level, there are several such rising and falling masses in a series stretching from west to east, the interchange of heat becomes more complicated. Thus the cold mass C will be found between two masses of warm air W , and the warm mass between two cold masses on the same horizontal level. In this case each warm mass W will divide and seek $C C$ on either side of it; the cold mass will also divide and seek $W W$ on either side. Since these small horizontal currents can not flow together from opposite directions to the center because a congestion of mass would occur, the motion is transformed into an inflowing helix with vertical component upward for a low pressure center L , and a counterflowing helix with downward vertical component with a high pressure H at the center of the vortex. This process is the cause of the minor whirls in the atmosphere, and contributes something to the formation of cyclones and anticyclones. In the latter case the warm and cold masses are not produced by vertical adiabatic changes, but by transportation of horizontal currents from great distances. The same tendency to divide the warm mass in the northern quadrants between the low and high pressure centers and to curl the cold mass into two branches in the southern quadrants of the high and low pressure areas, has been already found in the observations of the stream lines and the distribution of the temperatures. The tendency to divide and

¹ This paper logically follows No. IV, in the Review for June, 1906, but its publication has been delayed.—EDITOR.

² See Monthly Weather Review, March, 1906, page 114.

³ See Monthly Weather Review, March, 1906, page 113.

curl about the respective branches is common to all mixing masses of different temperatures.

FIG. 19.—Scheme of the transformation of adiabatic gradients into observed temperature gradients thru the heat terms $(Q - Q_0)$ and velocity terms $[(q^2 - q_0^2)]$.

$A E$ = Observed nonadiabatic gradient.

$A C$ = Adiabatic gradient for warm rising air.

$E W$ = Adiabatic gradient for cold descending air.

$C D$ = Quantity of heat $+ Q_1$ to be added to restore the equilibrium at the height $z - z_0$.

$W D$ = Quantity of heat $- Q_0$ to be lost in restoring the equilibrium.

At the level $C D W$ other amounts of heat, $+ \Delta Q_1$, $- \Delta Q_0$, are expended in setting up a velocity q_0 which is converted into a vortex with a vertical component.

If we find an adiabatic rate of temperature fall in the Tropics such as $10.0^\circ \text{ C. per } 1000 \text{ meters}$, but one of 5.0° C. in the temperate zones, and of only 2.0° C. in the polar zones, then this distribution between the Tropics and the polar zones is maintained by circulation and heat interchange. The streams of warm air in the lower strata, 0 to 3000 meters, and in the upper strata, 10,000 to 14,000 meters, on moving from the Tropics to cooler latitudes, gradually lose heat by expending the energy thru a series of minor and major gyrations which are set up. These streams near the surface tend by their rising to higher levels, as they approach the polar zones, to stratify the warmer air higher up in proportion to their departure from the Tropics, and thus to lessen the temperature fall from the surface; likewise, above the 10,000-meter level the same phenomenon occurs. Similarly, the cold polar currents flowing toward the equator tend to sink to lower levels, and this diminishes the temperature gradient in the middle latitudes. These two systems of currents can not traverse the space between the Tropics and the polar zones without encountering one another, and interacting upon each other, in the cyclones and anticyclones, and the general effect of the entire process is to maintain a gradient of temperature which differs from the adiabatic rate. The divergence between the actual and the adiabatic rate is very different from place to place, as shown by the observations. There is an incessant turmoil of adjustment at all levels; and in all latitudes, whose outcome is the wind, clouds, rain, and temperature actually prevailing. As above stated, it seems to be impossible to treat this physical complex as an analytic unit in the present state of meteorology, and hence I shall confine my discussion to a series of more or less detached studies, which yet tend to elucidate the general problem.

THE HORIZONTAL CIRCULATION.

In Tables 29 and 30,⁴ under the columns $z - z_0$, are given the vertical distances thru which the cold masses must fall and the warm masses rise, in order to attain an equilibrium on their respective levels. Thus, for the maximum cold masses in the east quadrant of the high area and the west quadrant of the low area, and for the maximum warm masses in the west quadrant of the high area and the east quadrant of the low area, we find the displacements in the winter, respectively, as follows:

TABLE 41.—Vertical displacement, $z - z_0$, from equilibrium.

Height in meters.	High east.	Low west.	Mean.	High west.	Low east.	Mean.
10000.....	+325	+454	+390	-325	-487	-406
9000.....	+366	+440	+403	-352	-440	-396
8000.....	+425	+467	+446	-425	-382	-404
7000.....	+498	+498	+498	-669	-356	-463
6000.....	+592	+568	+578	-740	-326	-533
5000.....	+748	+650	+699	-926	-325	-626
4000.....	+793	+646	+720	-1033	-333	-683
3000.....	+854	+726	+790	-1024	-299	-662
2000.....	+984	+777	+881	-1036	-311	-674
1000.....	+836	+593	+715	-890	-351	-621
0.....	+511	+414	+463	-487	-268	-378

The sign (+) means that the mass is too high by the given number of meters for thermodynamic equilibrium, and the sign (−) that the mass is too low. The cold masses can fall thru $z - z_0$ meters and the warm masses can rise thru $z - z_0$ meters on their respective levels, under the given conditions.

FIG. 20.—The conversion of vertical falls into horizontal circulation.

Thus at the 4000-meter level the cold mass can fall about 720 meters and the warm mass can rise 683 meters to bring about thermal equilibrium, when there is no horizontal circulation. If the cold air could sink to the level of the warm mass thermally, it would have a potential fall of 1403 meters, supposing this warm mass to remain unchanged in position and energy. The tendency is then for the cold mass in seeking the lowest thermal level not to fall vertically, but in the main to move almost horizontally down a gradient defined by $C W$. Assuming that the distance between the maxima C and W averages as in the ordinary cyclone about 1000 kilometers, or 1,000,000 meters we have a possible gradient,

$$G = \tan^{-1} \left(\frac{1403}{1,000,000} \right) = \tan^{-1} (0.001403) = 0^\circ 5' 3''.$$

As this large gradient would give very rapid horizontal motions there is too much power to be expended in this simple manner. The warm mass is really rising and the cold mass falling, simultaneously, not vertically but toward each other in the manner indicated by the diagrams, figs. 9, 10.⁵ In these, and from other descriptions of the prevailing circulation found in the reports of the Weather Bureau, we infer that in all of the levels the cold current flows southeastward toward the warm area, while the warm current flows northwestward toward the cold area.

This flow is not directly toward the respective centers of the warm and cold waves, making the currents meet along an axis,

⁴ See Monthly Weather Review for June, 1906, pp. 267-271.

⁵ See Monthly Weather Review, February, 1906, pp. 77-78 and lithograph plate at the end.

because this would produce a congestion of the density and make the flow impossible. The system of internal reactions in the circulating fluid, in combination with the deflecting force due to the earth's rotation, will cause the stream lines to flow about the center up to a certain limited amount of congestion on the outer circles. It is evident that a compromise or resultant between these opposite tendencies must be brought about, and then the stream lines will approximate to spirals converging toward the center in the cyclone, but diverging in the anticyclone. In order to avoid the congestion, a vortex motion is thus established with an ascending component over all areas contained within the closed isobars of the cyclone, but descending in the anticyclone. The conflict of this localized circulation with the general circulation, the continuous absorption of the former by the latter, produces the entire observed cyclone system. Quite similar reasoning accounts for the downward component in the anticyclone, which is generated and fed from the other portions of the cold and warm areas, since it has been shown that both of these masses divide into two branches and are absorbed in consecutive high and low pressure areas.

FIG. 21.—Scheme of the horizontal circulation in cyclones and anticyclones.

In the low area, in the strata from the surface to about 4000 meters, to the southward of the center, the cold mass tends to under-run the warm mass, while to the northward of the center in the strata above 4000 meters, the warm mass tends to overflow the cold mass. On the other hand, in the high pressure area, similar conditions exist tho the sectors or quadrants are inverted in their order. The cold air near the surface separates or divides into two branches, which tend to under-run the warm areas on either side, and in the high levels the warm air divides into two branches which tend to overflow the adjacent cold masses on either side.

FIG. 22.—Illustrating the relation of the thermodynamic gradients to the hydrodynamic pressures in cyclones and anticyclones.

In the warm areas the isobars are farther apart than in the cold areas, and by the ordinary rules the circulations are in the directions indicated. The warm mass divides into two branches which overflow the cold masses to the north, while the cold mass divides into two branches which under-run the warm masses to the south. The outcome is to produce more stable equilibrium by superposing air of less potential density upon air of greater potential density. At the same time there is an interchange of heat and a manifestation of dynamic energy in the form of large and small vortices on the horizontal planes with dynamic components in the vertical directions. In this process there are involved: (1) an interchange of heat; (2) a more stable equilibrium, since gravity has pulled the air of great potential density downward, while that of lower potential density is pushed up; (3) an amount of kinetic energy corresponding to the movements of the air masses from one level surface to another; (4) important horizontal motions with minor vortex motions whose kinetic energy represents a large fraction of that mentioned in the preceding item.

THE HORIZONTAL PRESSURE GRADIENTS.

In order that the reason for this overflow of warm masses upon cold masses in the upper strata, with underflow of cold masses beneath warm masses in the lower strata may be evident, we need only compute the pressures B in the several strata of the warm and cold masses, respectively, from the surface up to 10,000 meters. Combine the temperatures given in Table 21⁶ thus: Take the mean of the temperatures of the east sector of the high area and the west sector of the low area for the mean temperature in the cold mass, and the mean of the temperatures of the west sector of the high area and the east sector of the low area for the mean temperature of the warm mass, on each of the 1000-meter levels. The result will be found in Table 43, Section II, and is transferred to the first column of the cold and warm masses in Table 42, and marked t .

The mean t of the successive strata gives the mean temperature of the air column, $\theta = \frac{t_z + t_{z-1}}{2}$, in the second column. This

is the argument for m in Table 91, International Cloud Report, and we may assume that the observed t is the virtual temperature, and that it includes the dry air and the vapor contents as they occur. With H the height and θ as arguments, the value of m is extracted. It is now necessary to assume some value of the pressure B at the surface in warm and cold areas, independent of any variation due to the circulation in the high and low areas, and I have taken two pressures, 10 millimeters different, as fairly representing known surface pressures under the prescribed conditions. Thus, for 770 millimeters in the cold mass, we shall have 760 millimeters in the warm mass, as the barometric pressure at the surface. Adopt these values, take $\log B$, 2.88649 in the cold area, 2.88081 in the warm area, add successively the m on the several levels, and then take the corresponding B_c , B_w . Comparing B_c with B_w it is seen that the cold area pressure is greater than the warm area pressure up to 4000 meters, and that the warm area pressure is greater than the cold area pressure above that level. Hence, cold air flows to warm areas below, while warm air flows to cold areas above 4000 meters, conforming to well-recognized principles.

We can compute the vertical distance thru which 1 millimeter of air extends in the several levels. Take the difference between the pressures in the successive 1000-meter levels, $B - B_0$, the second difference, $\Delta (B - B_0)$, showing the variation with the height, then divide 1000 by $B - B_0$ for Δz , the required height in meters thru which 1 millimeter of air, that is the weight of air measured by 1 millimeter of mercury, extends. It changes from 11 meters near the surface to 31 meters near the 10,000-meter level, and shows the spaces that exist in a vertical direction between successive isobaric surfaces.

⁶ Page 268, Monthly Weather Review, June, 1906.

Since the tendency of gravity is to make these spaces equal in the same stratum, a circulation is set up to bring this about; this is the flowing of the air which, thereupon, builds up the observed cyclones and anticyclones in combination with the other forces, inertia, expansion and contraction, deflection, centrifugal, friction, and internal vortical motion. This complex network of forces can be reduced to a rigid analytic discussion only with the greatest difficulty, even without the term involving the interchange of heat energy into velocity, and it seems nearly useless to attempt it until further experimental knowledge of this process in the free air has been obtained by a careful discussion of the temperature conditions observed in balloon and kite ascensions.

TABLE 42.—*Computation of the pressure B in the cold and warm maxima on each 1000-meter level.*

Height in meters.	In cold masses.					In warm masses.				
	<i>t</i>	θ	<i>m</i>	<i>B_c</i>	<i>B_e</i>	<i>t</i>	θ	<i>m</i>	<i>B_w</i>	<i>B_e</i>
	$^{\circ}\text{C.}$	$^{\circ}\text{C.}$		<i>log.</i>	<i>mm.</i>	$^{\circ}\text{C.}$	$^{\circ}\text{C.}$		<i>log.</i>	<i>mm.</i>
10000...	-56.6	-53.6	6796	2.28423	192.41	-51.7	-48.4	6595	2.29445	196.99
9000...	-50.5	-47.2	6561	2.35219	225.01	-45.0	-41.4	6397	2.36040	229.30
8000...	-43.8	-40.5	6371	2.41780	261.70	-37.8	-34.1	6200	2.42437	265.69
7000...	-37.1	-33.9	6195	2.48151	303.05	-30.4	-26.8	6016	2.48637	306.46
6000...	-30.6	-27.5	6033	2.54346	349.51	-23.1	-19.7	5849	2.54653	351.99
5000...	-24.3	-21.3	5885	2.60379	401.60	-16.2	-13.4	5705	2.60502	402.74
4000...	-18.2	-15.5	5752	2.66264	459.86	-10.6	-8.3	5595	2.66207	459.27
3000...	-12.7	-10.2	5636	2.72016	525.00	-5.9	-3.7	5499	2.71802	522.42
2000...	-7.6	-5.5	5536	2.77652	597.75	-1.6	0.0	5425	2.77301	592.94
1000...	-3.3	-0.8	5461	2.83188	679.02	+1.7	+3.5	6355	2.82726	671.83
0...	+1.7			2.88649	770.00	+5.2			2.88081	760.00

Vertical distance for 1 mm. of pressure between strata of different temperature.

Height.	$B-B_0 \Delta (B-B_0) (B-B_0) \Delta_z \Delta_z$					$B-B_0 \Delta (B-B_0) (B-B_0) \Delta_z \Delta_z$				
	<i>mm.</i>	<i>mm.</i>	<i>log.</i>	<i>log.</i>	<i>mm.</i>	<i>mm.</i>	<i>log.</i>	<i>log.</i>	<i>mm.</i>	<i>mm.</i>
10000...	32.60	4.09	1.51322	1.48678	30.68	32.31	1.50934	1.49066	30.95	
9000...	36.69	4.66	1.56455	1.43545	27.25	36.39	1.56098	1.43902	27.48	
8000...	41.35	5.11	1.61648	1.38352	24.18	40.77	1.61034	1.38966	24.53	
7000...	46.46	5.63	1.66708	1.33291	21.52	45.53	1.65830	1.34170	21.96	
6000...	52.09	6.17	1.71675	1.28325	19.20	50.75	1.70544	1.29456	19.70	
5000...	58.26	7.08	1.76537	1.23463	17.16	56.53	1.75228	1.24772	17.69	
4000...	65.14	7.61	1.81385	1.18615	15.35	63.15	1.80037	1.19963	15.84	
3000...	72.75	8.52	1.86183	1.13817	13.75	70.52	1.84831	1.15169	14.18	
2000...	81.27	9.71	1.90993	1.09007	12.30	78.89	1.89702	1.10298	12.68	
1000...	90.98		1.95895	1.04105	10.99	88.17	1.94532	1.05468	11.34	
0...										

THE HORIZONTAL INTERCHANGE OF HEAT ENERGY.

We can secure some idea of the process involved in the interchange of the heat energy on the horizontal surfaces by a computation of the formula:

Term I Term II
 $C_p n_0 (T_1 - T) + C_p T_0 \log T_0 (n_1 - n) = (Q_1 - Q) - \frac{1}{2} (q_1^2 - q^2)$.
 The necessary data are collected in Table 43, and they are gathered in the same way as described for the temperatures, by combining the sectors of cold and of warm masses, respectively. The mean value of the gradient ratio *n* is found by extracting *n* from Tables 25 and 26, and taking the means, *n* for cold areas and *n*₁ for warm areas. Then the difference, *n*₁ - *n*, and the mean, *n*₀ = $\frac{1}{2} (n + n_1)$, are taken out for use in the formula. We adopt the notation (*n*, *t*, *q*, *Q*) for the cold mass, (*n*₁, *t*₁, *q*₁, *Q*₁) for the warm mass, and (*n*₀, *t*₀, *q*₀, *Q*₀) the mean values of the cold and warm masses when required.

TABLE 43.

I.—Mean values of the gradient ratio *n* in the cold and warm maxima.

Height in meters.	Ratio. <i>n</i>			Ratio. <i>n</i> ₁			<i>n</i> ₁ - <i>n</i> W-C.	Mean <i>n</i> ₀
	High east.	Low west.	Mean cold.	High west.	Low east.	Mean warm.		
10000.....	1.778	1.659	1.718	1.535	1.547	1.541	-177	1.630
9000.....	1.537	1.500	1.518	1.319	1.458	1.389	-129	1.454
8000.....	1.495	1.443	1.469	1.246	1.447	1.347	-122	1.408
7000.....	1.523	1.447	1.485	1.234	1.471	1.353	-132	1.419
6000.....	1.567	1.498	1.533	1.272	1.518	1.395	-138	1.464
5000.....	1.623	1.562	1.593	1.430	1.629	1.530	-063	1.562
4000.....	1.725	1.690	1.708	1.974	1.766	1.870	+162	1.789
3000.....	1.894	1.876	1.885	2.443	1.974	2.209	+324	2.047
2000.....	2.285	2.150	2.218	3.056	2.518	2.787	+569	2.503
1000.....	2.179	2.213	2.196	3.439	2.611	3.025	+829	2.611
000.....	1.769	2.065	1.917	3.290	2.367	2.829	+912	2.373

II.—Mean values of the temperature *T* in the cold and warm maxima.

Height in meters.	Temperature. <i>t</i>			Temperature. <i>t</i> ₁			<i>t</i> ₁ - <i>t</i> W-C.	Mean <i>T</i> ₀ <i>t</i> ₀ + 273°	Log. <i>T</i> ₀
	High east.	Low west.	Mean cold.	High west.	Low east.	Mean warm.			
	$^{\circ}\text{C.}$	$^{\circ}\text{C.}$	$^{\circ}\text{C.}$	$^{\circ}\text{C.}$	$^{\circ}\text{C.}$	$^{\circ}\text{C.}$	$^{\circ}\text{C.}$	<i>Abs.</i>	
10000.....	-56.2	-57.0	-56.6	-52.2	-51.2	-51.7	+4.9	218.8	2.84005
9000.....	-50.2	-50.7	-50.5	-45.3	-44.7	-45.0	+5.5	225.2	2.35257
8000.....	-43.6	-43.9	-43.8	-37.6	-37.9	-37.8	+6.0	232.2	2.86586
7000.....	-37.1	-37.1	-37.1	-29.6	-31.1	-30.4	+6.7	239.2	2.37876
6000.....	-30.7	-30.5	-30.6	-21.7	-24.5	-23.1	+7.5	246.1	2.39111
5000.....	-24.6	-24.0	-24.3	-14.3	-18.0	-16.2	+8.1	152.7	2.40261
4000.....	-18.6	-17.8	-18.2	-8.7	-12.5	-10.6	+7.6	258.6	2.41263
3000.....	-13.0	-12.4	-12.7	-4.2	-7.6	-5.9	+6.8	263.7	2.42111
2000.....	-8.0	-7.2	-7.6	-0.2	-3.0	-1.6	+6.0	268.4	2.42878
1000.....	-3.7	-2.8	-3.3	+2.7	+0.7	+1.7	+5.0	272.2	2.43489
000.....	+1.5	+1.9	+1.7	+5.6	+4.7	+5.2	+3.5	276.5	2.44170

III.—Mean values of the velocity term in the cold and warm maxima.

Height in meters.	Velocity. $\frac{1}{2}(q_1^2 - q^2)$			Velocity. $\frac{1}{2}(q_1^2 - q^2)$			Average.
	High west.	Low east.	Mean warm.	High east.	Low west.	Mean cold.	
10000.....	+54	+175	+115	-127	-95	-111	113
9000.....	+56	+216	+136	-122	-122	-122	129
8000.....	+72	+228	+150	-101	-132	-117	134
7000.....	+72	+220	+146	-91	-121	-106	126
6000.....	+65	+160	+113	-83	-83	-83	98
5000.....	+88	+88	+88	-74	-50	-62	75
4000.....	+72	+72	+72	-56	-32	-44	58
3000.....	+60	+86	+73	-58	-14	-36	55
2000.....	+10	+52	+31	-28	-28	-28	30
1000.....	+3	+19	+11	-12	-25	-19	15
000.....	+8	+18	+13	-8	-8	-8	11

These data are given in Section I of Table 43; the temperature data in Section II of that table are taken from Tables 21 and 22; *T*₀ and log *T*₀ are computed; finally, $\frac{1}{2} (q_1^2 - q^2)$ are taken from Tables 33 and 34. Since the velocity energy is a small term in comparison with (*Q*₁ - *Q*), there is no need to be particular about the exact velocities, and approximate values are sufficient. In order to learn the relation between the values of the ratio *n*, *n*₁ in cold and warm areas in the

several strata, they are plotted in fig. 23. It is seen that the curves cross each other between the 4000 and the 5000-meter level, showing that there is a reversal of the physical process at that elevation, as warming below and cooling above, so that the cold mass is warming below and the warm mass is cooling above in conformity with the preceding statements. Since the adiabatic gradient is -9.87° C. per 1000 meters, and $a = \frac{a_0}{n}$, we find the gradients corresponding with n at the several levels by using the lower horizontal argument in the diagram.

FIG. 23.—Mean values of the gradient ratio, n , at the cold and warm maxima.

The computation of the terms $I = C_p n_0 (T_1 - T)$ and $II = C_p T_0 \log T_0 (n_1 - n)$ gives the results that are found in Table 44, for the several 1000-meter levels. Term I is positive for all levels, and term II reverses the sign at about the 5000-meter level. The sum $I + II$ is reduced to calories by the factor $A_m = 0.0002389$ in Table 14.⁷ In the last column of Table 44, a mean value of $\frac{1}{2} (q_1^2 - q_0^2)$ is added as computed by Section III, Table 43. A comparison of columns 4 and 6 shows how small the velocity term is in comparison with the heat term. An unknown (R) is added in the formula to represent the waste of energy in passing thru friction into motion. It stands between the energy and velocity terms, but can not be evaluated, and it is presupposed in the unexpressed function that connects heat with motion. In the same way there is the unknown radiation term, J , wherein some heat energy is wasted so far

as the motion of the atmosphere is concerned. The function uniting $(Q_1 - Q) - \frac{1}{2} (q_1^2 - q_0^2) + (R) + (J)$ being undetermined, it is very difficult to make satisfactory progress in this direction, and the problem must wait for further developments. Reviewing columns I + II in calories, which is the heat energy available from the temperature distribution, it is seen that it is positive and diminishes up to the 5000-meter level, above which it is small and negative. Comparing this column with Tables 37 and 38 it is observed that the vertical heat potentiality is about the same as the horizontal capacity for motion. If a kilogram of air is moved as noted by the conditions of the problem, this amount of heat must be interchanged. In the actual atmosphere this transfer is not so simple, and hence only a portion of the Q -energy is actually produced. How much less is really generated depends upon the efficiency of the thermodynamic engine in the practical physical operations of the air.

TABLE 44.—Values of the terms in the formula,

$$I = C_p n_0 (T_1 - T) + C_p T_0 \log T_0 (n_1 - n) \\ = (Q_1 - Q) - \frac{1}{2} (q_1^2 - q_0^2) + R + J.$$

Energy terms in the horizontal convection.

Height in meters.	I	II	I + II	I + II in calories.	$\frac{1}{2} (q_1^2 - q_0^2)$
10000.....	7936	-90042	-82106	- 19.6	113
9000.....	7946	-67905	-59959	- 14.3	129
8000.....	8394	-66587	-58193	- 13.9	134
7000.....	9443	-74624	-65181	- 15.6	126
6000.....	10909	-80684	-69775	- 16.7	98
5000.....	12571	-38004	-25433	- 6.1	75
4000.....	13509	100427	113936	+ 27.2	58
3000.....	13830	205529	219359	+ 52.4	55
2000.....	14921	368533	383454	+ 91.6	30
1000.....	12971	545913	558884	+133.5	15
0.....	8252	611771	620023	+148.1	11

SOME CASES OF RESTRICTED CONDITIONS.

In order to approach this intricate problem by a mathematical analysis, it will be desirable to study some simpler cases, or models, wherein the conditions are limited by ideal restrictions. These consist in placing two masses of air in adjoining chambers, or in one chamber with a movable partition, whereby two fixed masses under given conditions when set into communication react upon each other. Dr. M. Margules has made several such studies in his paper, *Über die Energie der Stürme*, and for the sake of profiting by this excellent work, I have prepared a brief synopsis of the results as modified by myself to meet nonadiabatic conditions. It is proposed to give the assumed data and the resulting formula, omitting the algebraic reductions, and to urge that the student should not fail to read that paper. In order to preserve the notation of my formula, the following table of equivalents will be useful:

Margules. Bigelow.

External kinetic energy	K to $(K) = \frac{1}{2} \int \rho q^2 dv = \frac{1}{2} m q^2.$
External potential energy	P to $V = \int \rho (-gr + \frac{1}{2} w_0^2 \omega^2) dv.$
Internal kinetic energy } Internal potential energy }	I to $U = \left\{ \frac{H_m}{J_m} (\text{molecules}) + \frac{H_a}{J_a} (\text{atoms}) \right\} = C_v \int T \rho dv.$
Quantity of heat	$Q = \int dt \int \frac{dQ}{dt} \rho dv.$
Work of expansion	A to $-W = - \int dt \int \frac{p}{\rho} \frac{d\rho}{dt} dv = \int dt \int p v \frac{1}{\rho} \frac{d\rho}{dt} dv.$

⁷ See Monthly Weather Review, March, 1906, page 115.

Margules. Bigelow.

Potential energy + centrifugal force W to	$V_1 = -gr + \frac{1}{2}w_0^2 \varpi^2$.
Friction	$(R) = - \int dt \int R q \cos(Rq) \rho dv$.
Velocity	c, V to q
Volume	k to v
Density	μ to ρ
Ratio of specific heats	γ to $k = \frac{C_p}{C_v}$.
Adiabatic constant	$\frac{1}{x}$ to $\frac{k}{k-1} = \frac{C_p}{R} = \frac{g_0}{R a_0}$.
Height	c to h
Surface	O to S
Entropy temperature	θ to T_0
Potential temperature	τ to T_0
Drive temperature	ϑ to T_0

GENERAL THERMODYNAMIC EQUATIONS.

- (1) Conservation of energy. $\left\{ \begin{array}{l} \delta(K) + \delta V - \delta W + (R) = 0. \\ Q = \delta U + \delta W + (R) = \delta(K) + \delta V + \delta U + (R). \\ Q = [\delta(K) + \delta V] \text{ external} + [\delta H + \delta J] \text{ internal} = \delta W + \delta U. \end{array} \right.$
- External work. Internal heat.
- (2) Variation of heat. $\left\{ \begin{array}{l} dQ = R T \frac{dv}{v} + \frac{1}{A} C_v dT. \\ dQ = -R T \frac{dp}{p} + \frac{1}{A} C_p dT. \\ dQ = p dv + \frac{v dp + p dv}{A R}. \end{array} \right. \text{ in mechanical units.}$
- (3) External potential energy. $\left\{ \begin{array}{l} V = \int_0^\infty g z \rho dz = \int_0^h g z \rho dz + g z M_h. \quad p_h = g M_h = g \rho h. \\ V = - \int_p^{p_h} z dp + Z p_h + \int_0^h p dz - z p_h + Z p_h. \\ V = \int_0^h p dz + (Z - z) p_h = R \int T dm + \text{const.} \end{array} \right.$
- (4) Internal energy. $U = C_v \int T dm + \text{const.}$
- $(U + V) = (C_v + R) \int T dm + \text{const.} = C_p \int T dm + \text{const.}$
- (5) Transformation of energy. $\left\{ \begin{array}{l} -\delta(U + V) = (U + V)_a(\text{initial}) - (U + V)_e(\text{final}) = C_p \int (T - T^1) dm. \\ \delta(K) + (R) = \frac{1}{2} M q^2 = C_p \int (T - T^1) dm = C_p (T - T^1) M. \end{array} \right.$
- (6) Entropy variations. $\left\{ \begin{array}{l} S - S_0 = \int \frac{dQ}{T} = C_v \log \frac{T}{T_0} + R \log \frac{v}{v_0}. \\ S - S_0 = \int \frac{dQ}{T} = C_p \log \frac{T}{T_0} - R \log \frac{p}{p_0}. \\ \frac{\partial S}{\partial z} = \frac{1}{T} \frac{\partial Q}{\partial z} = \frac{C_p}{T} \frac{\partial T}{\partial z} - \frac{R}{p} \frac{\partial p}{\partial z}. \end{array} \right.$
- (7) Potential temperature. $T_0 = T \left(\frac{P_0}{P} \right)^{\frac{k-1}{k}}.$
- (8) In linear vertical changes. $\int_0^h T dm = \frac{1}{g} \frac{1}{1 + \frac{k-1}{nk}} (p_n T_0 - p T).$

(9) Auxiliary equations.

$$\left\{ \begin{array}{ll} \frac{\rho}{\rho_0} = \left(\frac{T}{T_0} \right)^{\frac{n}{k-1}} = \left(\frac{P}{P_0} \right)^{\frac{1}{k}} = \frac{v_0}{v}. \\ \frac{P}{P_0} = \left(\frac{T}{T_0} \right)^{\frac{nk}{k-1}} = \left(\frac{\rho}{\rho_0} \right)^k = \left(\frac{v_0}{v} \right)^k. \\ \text{Adiabatic.} & \text{Observed.} \\ \frac{k}{k-1} = \frac{C_p}{R} = \frac{g}{Ra_0}. & \frac{nk}{k-1} = \frac{nC_p}{R} = \frac{g}{Ra}. \\ \frac{1}{\rho} = \frac{1}{P} R T. & \frac{1}{\rho} \frac{dP}{dz} = -g. \end{array} \right. \quad a = \frac{a_0}{n} = \frac{g}{nC_p}.$$

$$\frac{1}{P} \frac{dP}{dz} = -\frac{1}{RT} g.$$

CASE I. CHANGE OF POSITION OF THE LAYERS IN A COLUMN OF AIR.

In consequence of the general and local circulations of the

FIG. 24 A. Initial.

Final.

atmosphere, a certain gradient $a = \frac{a_0}{n}$ prevails at a given locality in a column above the earth's surface. This requires an amount of heat Q_0 and a temperature T_0 at each level z_0 to maintain the stratum in equilibrium. If the heat energy changes to Q for any reason or the temperature is altered to T there must follow a change in elevation to z to restore the equilibrium. The equation of equilibrium,

$$(10) \quad \frac{1}{2} (q^2 - q_0^2) = (Q - Q_0) - C_p n (T - T_0) - C_p T_0 \log \frac{T_0 (n - n_0) - g(z - z_0)}{T_0 (n - n_0) - g(z - z_0)},$$

is available for the computation of the motion due to stratifications in the column. In order to take a simple case we assume that each air mass retains its own heat energy or $Q = Q_0$, and that the gradient is the same thruout the column or $n = n_0$. Hence when starting from rest or $q = 0$, the equation becomes for the unit mass.

$$(11) \quad \frac{1}{2} q^2 = -C_p n (T - T_0) - g(z - z_0).$$

This must be applied to each mass moved, so that finally

$$(12) \quad \frac{1}{2} m q^2 = \sum \left[-C_p n (T - T_0) - g(z - z_0) \right] m.$$

Let the column be separated from the surrounding air by walls and consist of four parts. M_0 is a lower section not affected by the transfer; the next layer m_1 , under pressure P_1 and temperature T_1 , is not in equilibrium, so that the stratified layer m_1 must rise if T_1 is too warm and fall if T_1 is too cold for its elevation z_1 . If it rises thru a height $h = z_2 - z_1$, and by expanding cools to a given temperature T_1' , the pressure P_1 will become P_h and be in equilibrium; the section M_2 of thickness h falls a certain distance and changes its temperature; for the upper differential layer dM_2 the initial values

P_2, T_2 , become P_2', T_2' , and the function must be integrated thruout the mass M_2 ; the temperature of the mass M_h is not affected by the mutual transfer of m_1, M_2 , but rises or falls like a piston in the chamber, while its lower surface maintains the pressure P_h . Hence, we have the conditions,

Layer.	Initial.	Final.	Pressure.
dM	P_2, T_2	P_2', T_2'	$P_2' = P_2 + g m_1$
m_1	P_1, T_1	P_h, T_1'	$P_1 = P_h + g M_2$

$$(13) \quad T_2' = T_2 \left(\frac{P_2'}{P_2} \right)^{\frac{k-1}{nk}} = T_2 \left(1 + \frac{g m_1}{P_2} \right)^{\frac{k-1}{nk}} = T_2 \left(1 + \frac{k-1}{nk} \frac{g m_1}{P_2} \right)$$

$$= T_2 + T_2 \frac{R}{nC_p} \frac{g m_1}{P_2}.$$

$$(14) \quad T_1' = T_1 \left(\frac{P_h}{P_1} \right)^{\frac{k-1}{nk}}.$$

Substituting in the equation,

$$(15) \quad \text{Kinetic energy} = C_p \left[\int (T_1 - T_1') dm_1 + \int (T_2 - T_2') dM_2 \right].$$

$$(16) \quad \frac{1}{2} m_1 q^2 = C_p \left[T_1 - T_1' \left(\frac{P_h}{P_1} \right)^{\frac{k-1}{nk}} \right] m_1 - g \frac{h}{n} m_1,$$

since

$$(17) \quad \frac{RT_2}{P_2} \int \frac{dM_2}{n} = \int \frac{dM_2}{n\rho_2} = \int \frac{dz}{n} = \frac{h}{n}.$$

The gravity terms in these equations disappear, because the mechanical work in each case, $g h M_1$ and $g (Z_2 - Z_1) M_2$ (where Z_2 is the height of the center of gravity of M_2) is of the same amount and oppositely directed. Every expansion or contraction of air masses begins on an adiabatic gradient, and hence the formulas must be founded on that basis. But minor interchanges of energy as heat Q and velocity $\frac{1}{2} q^2$ almost immediately begin in the mixing process, so that the theoretical conditions soon suffer modifications which it is quite impracticable to follow out.

CASE II. THE TEMPERATURE IS A CONTINUOUS FUNCTION OF THE HEIGHT,
 $T_2 = T_1 - a h.$

It is important to eliminate the pressures from the formula and express the function in terms of g, h, T , and the gradients. Several forms of the function for the temperature distribution may be employed to represent the atmosphere, but it is only occasionally that these formulas can be used to replace the actual pressure and temperature observations at different levels. For the observed gradient we have

$$(18) \quad \text{Observed } \left\{ \begin{array}{l} \frac{P_h}{P_1} = \left(\frac{T_2}{T_1} \right)^{g/Ra} = \left(\frac{T_1 - ah}{T_1} \right)^{g/Ra}. \\ \text{gradient.} \end{array} \right.$$

Hence,

$$(19) \quad \left. \begin{array}{l} \text{Adiabatic} \\ \text{gradient.} \end{array} \right\} \left(\frac{P_h}{P_1} \right)^{\frac{k-1}{k}} = \left(1 - \frac{ah}{T_1} \right)^{g/Ra \frac{k-1}{k}} = \left(1 - \frac{ah}{T_1} \right)^{g/C_p a}$$

Then,

$$(20) \quad C_p m_1 (T_1 - T_1^1) = C_p m_1 \left(T_1 - T_1 + \frac{gh}{C_p} - \frac{1}{2} \frac{g^2 h^2}{C_p^2 T_1} + \frac{1}{2} \frac{g h^2 a}{C_p T_1} \right).$$

Finally,

$$(21) \quad \frac{1}{2} m_1 q^2 = gh m_1 - \frac{1}{2} \frac{g h^2}{C_p T_1} \cdot \frac{g}{C_p} + \frac{1}{2} \frac{g h^2}{C_p T_1} \cdot a - gh m_1 \\ = \frac{1}{2} \frac{g h^2}{T_1} m_1 \left(a - \frac{g}{C_p} \right), \\ = \frac{1}{2} \frac{g h^2}{T_1} m_1 \left(\frac{a_0}{n} - a_0 \right).$$

The mass m_1 is driven from its position with a velocity-energy inversely proportional to the temperature, so that warm air has less driving power than cold air. The drive depends upon the departure-ratio n and vanishes when $n=1$, that is, for an adiabatic expansion in an adiabatic gradient. When $a > a_0$ the mass m_1 is in unstable equilibrium—is too cold for its position and tends to fall. Example, for $n=0.5$, $a=19.74 > a_0=9.87$. When $a < a_0$ the mass m_1 is in stable equilibrium. Example, for $n=2$, $a=4.94 < a_0=9.87$. It is not possible to drive the small mass m_1 thru any great height h in the atmosphere, because the differential energy in the expanding mass sets up minor whirls which tend to interchange the Q -energy by mechanical effects and internal friction.

The result is to change the gradient from a_0 to $a = \frac{a_0}{n}$. If the displacement of the mass m_1 takes place in the medium of gradient a then the drive may be expressed by terms of the form,

$$(22) \quad \frac{1}{2} g \frac{h^2}{T_1} m_1 \left(\frac{a_0}{n_1} - \frac{a_0}{n} \right) = \frac{1}{2} \frac{gh^2}{T_1} a_0 \left(\frac{n - n_1}{n n_1} \right),$$

where n_1 is the effective ratio of the moving mass m_1 and a that of the prevailing general gradient.

AUXILIARY THEOREM. EVALUATION OF $\int T dm$ IN LINEAR VERTICAL TEMPERATURE CHANGES.

$$(25) \quad \text{Assume } T = T_0 - az, \quad P = P_0 \left(\frac{T}{T_0} \right)^{g/Ra}, \quad \int T dm = \int T \rho dz.$$

$$\int_0^z T \rho dz = \frac{1}{R} \int_0^z P dz = \frac{1}{R} \int_0^z P_0 \left(\frac{T}{T_0} \right)^{g/Ra} dz = \frac{1}{R} \int_0^z P_0 T^{g/Ra} T_0^{-g/Ra} dz$$

Change the limits of integration from z to T .

$$(26) \quad T = T_0 - az, \quad dT = -a dz, \quad -\frac{1}{a} dT = dz, \quad \int_0^z T^x dz = -\frac{1}{a} \int_{T_0}^T T^x dT.$$

$$(27) \quad \int_0^z T \rho dz = \frac{1}{Ra} P_0 T_0^{-g/Ra} \int_T^{T_0} T^{g/Ra} dT = \frac{1}{Ra} P_0 T_0^{-g/Ra} \left[\frac{T_0}{T} \frac{1}{1+g/Ra} T^{g/Ra+1} \right] \\ = \frac{1}{g+Ra} P_0 T_0^{-g/Ra} \left(T_0^{g/Ra+1} - T^{g/Ra+1} \right) = \frac{1}{g+Ra} (P_0 T_0 - P T).$$

For any gradient other than the adiabatic we have,

$$(28) \quad \int_0^z T \rho dz = \frac{1}{g} \frac{1}{1 + \frac{k-1}{n k}} (P_0 T_0 - P T).$$

CASE III. FOR LOCAL CHANGES BETWEEN TWO ADJACENT STRATA OF DIFFERENT TEMPERATURES, WHERE ON THE BOUNDARY THE PRESSURE $P = P_1^1 = P_2^1$, AND THE TEMPERATURE IS DISCONTINUOUS.

Take the following conditions:

Layer.	Initial.	Final.	Pressure.	Temperature.
m_2	$P_2 T_2$	$P_2^1 T_2^1$	$P_2^1 = P_2 + gm_1$	$T_2^1 = T_2 \left(\frac{P_2 + gm_1}{P_2} \right)^{\frac{k-1}{n k}}$
m_1	$P_1 T_1$	$P_1^1 T_1^1$	$P_1^1 = P_1 - gm_2$	$T_1^1 = T_1 \left(\frac{P_1 - gm_2}{P_1} \right)^{\frac{k-1}{n k}}$

The equation of equilibrium becomes, for $P_1^1 = P_2^1 = P$,

$$(23) \quad \text{Kinetic} = C_p \left[m_1 (T_1 - T_1^1) + m_2 (T_2 - T_2^1) \right], \\ \text{energy} \\ = C_p \left[m_1 (T_1 - T_1 + T_1 \frac{R}{n} \frac{g m_2}{C_p P}) \right. \\ \left. + m_2 (T_2 - T_2 - T_2 \frac{R}{n} \frac{g m_1}{C_p P}) \right], \\ = m_1 m_2 \frac{R g}{n P} (T_1 - T_2), \\ = m_1 m_2 \frac{g}{n} \left(\frac{1}{\rho_1} - \frac{1}{\rho_2} \right).$$

Since $\frac{R T_1}{P_1} = \frac{1}{\rho_1}$ and $\frac{R T_2}{P_2} = \frac{1}{\rho_2}$, therefore

$$(24) \quad \frac{1}{2} M q^2 = m_1 m_2 \frac{g}{n} \frac{\rho_2 - \rho_1}{\rho_1 \rho_2}.$$

The kinetic energy inducing an interchange is proportional to the difference of the densities and inversely proportional to the product of the densities. Hence, if strata of different densities are flowing over one another in the general circulation which is temporarily stratified, these two strata tend to mix by interpenetration according to this law.

CASE IV. THE OVERTURN OF DEEP STRATA IN THE COLUMN.

Let the pressures, temperatures, and heights be arranged in the initial and final states as indicated in the diagrams (fig. 24 B). The greatest entropy in 1 is less than the least in 2, so that the cold mass 1 will fall beneath the warm mass 2. The heights of the masses will change as well as the pressures and temperatures.

Assume P_0 , T_{02} , h_2 , T_{i1} , h_1 , as known in the initial state.

Pressures.

Temperatures.

$$(29) \quad P_i = P_0 \left(\frac{T_{i2}}{T_{02}} \right)^{\frac{nk}{k-1}}. \quad T_{i2} = T_{02} - \frac{g h_2}{n C_p}.$$

$$(30) \quad P_h = P_i \left(\frac{T_{h1}}{T_{i1}} \right)^{\frac{nk}{k-1}}. \quad T_{h1} = T_{i1} - \frac{g h_1}{n C_p}.$$

FIG. 24 B.

Substitute in $C_p \left(\int T dm - \int T_1 dm_1 \right)$ successively.

$$(31) \quad \text{Initial, } (V + U)_a = C_p \int T dm = \frac{C_p}{g} \frac{1}{1 + \frac{k-1}{nk}} (P_0 T_{02} - P_i T_{i2} + P_i T_{i1} - P_h T_{h1}) + \text{const.}$$

$$(32) \quad \text{Final, } (V + U)_e = \frac{C_p}{g} \frac{1}{1 + \frac{k-1}{nk}} (P_0' T_{01}' - P_i' T_{i1}' + P_i' T_{i2}' - P_h T_{h2}') + \text{const.}$$

$$(33) \quad \text{Kinetic energy} = (V + U)_a - (V + U)_e = \frac{1}{2} M q^2 = \frac{1}{2} \frac{P_0 - P_h}{g} q^2.$$

$$(34) \quad \text{Heights, } h_1' = \frac{n C_p}{g} (T_{01}' - T_{i1}'), \quad h_2' = \frac{n C_p}{g} (T_{i2}' - T_{h2}').$$

$$(35) \quad \text{Approximate solution of Case IV.} \quad \frac{1}{2} q^2 = \frac{g h_1 h_2}{n h_1 T_{i2} + h_2 T_{i1}}.$$

CASE V. TRANSFORMATION OF TWO MASSES OF DIFFERENT TEMPERATURES ON THE SAME LEVEL INTO A STATE OF EQUILIBRIUM.

FIG. 24 C.

Given as data at the height h , T_{h1} , T_{h2} , P_h , the areas B_1 , B_2 , the entropy $S_1 < S_2$. Hence by the formulas,

$$(36) \quad P_{01} = P_h \left(\frac{T_{01}}{T_{h1}} \right)^{\frac{nk}{k-1}} = P_h \left(1 + \frac{g h}{n C_p T_{h1}} \right)^{\frac{nk}{k-1}}. \quad T_{01} = T_{h1} \left(1 + \frac{g h}{n C_p T_{h1}} \right).$$

$$(37) \quad P_{02} = P_h \left(\frac{T_{02}}{T_{h2}} \right)^{\frac{nk}{k-1}} = P_h \left(1 + \frac{g h}{n C_p T_{h2}} \right)^{\frac{nk}{k-1}}. \quad T_{02} = T_{h2} \left(1 + \frac{g h}{n C_p T_{h2}} \right).$$

$$(38) \quad \text{Initial. } (V + U)_a = C_p \frac{1}{g} \frac{1}{1 + \frac{k-1}{nk}} \frac{B}{2} (P_{01} T_{01} - P_h T_{h1} + P_{02} T_{02} - P_h T_{h2}) + \text{const.}$$

$$(39) \quad P_i' = P_h + \frac{1}{2} (P_{02} - P_h). \quad P_0' = P_h + \frac{1}{2} (P_{02} - P_h) + \frac{1}{2} (P_{01} - P_h).$$

$$(40) \quad \text{Final. } (V + U)_e = C_p \frac{1}{g} \frac{1}{1 + \frac{k-1}{nk}} B (P_0' T_0' - P_i' T_{i1}' + P_i' T_{i2}' - P_h T_{h2}) + \text{const.}$$

$$(41) \quad \text{Kinetic energy. } \frac{1}{2} M q^2 = (V + U)_a - (V + U)_e.$$

$$(42) \quad \text{Mass and heights. } M = \frac{B}{g} (P_0' - P_h). \quad h_1' = \frac{C_p}{g} (T_0' - T_{i1}'). \quad h_2' = \frac{C_p}{g} (T_{i2}' - T_{h2}).$$

(43) Approximate solution for Case V. Take $\tau = \frac{T_2 - T_1}{T}$. $T^2 = T_1 T_2$. $M = B P_h \frac{h}{R T} = B \rho h$ (approximate).

$$(44) \quad \frac{1}{2} M q^2 = \frac{1}{2} M \frac{B_1 B_2}{B^2} g h \tau.$$

CASE VI. CONTINUOUS HORIZONTAL TEMPERATURE DISTRIBUTION WITH ADIABATIC VERTICAL GRADIENT.

FIG. 24 D.

$$(45) \text{ Assume } T = f(x) - \frac{g}{C_p} z.$$

$$(46) \quad P^1 = P_h + \frac{1}{l} \int_{l-x}^l (P_{0x} - P_h) dx = P - \left[P - P_h - \frac{1}{l} \int_{l-x}^l (P_{0x} - P_h) dx \right].$$

$$(47) \quad T - T^1 = T - T \left(\frac{P^1}{P} \right)^{\frac{k-1}{k}} = \frac{k-1}{k} \left(T - \frac{P_h}{R \rho} - \frac{1}{l R \rho} \int_{l-x}^l (P_{0x} - P_h) dx \right) dx.$$

$$(48) \quad T_x \frac{P_{0x} - P_h}{g} = T_x \int_0^h \rho dz = \int_0^h T \rho dz.$$

$$(49) \quad \int_0^h (T - T^1) \rho dz = \frac{k-1}{k} \frac{1}{g} P_h T_0 \left(\frac{gh}{R T_0} \right)^2 \left(\frac{1}{2} - \frac{x}{l} + \frac{\tau x}{2l} - \frac{\tau x^2}{2l^2} \right).$$

$$(50) \quad \frac{1}{2} M q^2 = C_p \int_0^h (T - T^1) dm = l P_h \frac{gh}{R T_0} h \frac{\tau}{12}.$$

$$(51) \quad q = \sqrt{\frac{gh\tau}{6}}.$$

CASE VII. POSITION OF LAYERS OF EQUAL ENTROPY WHEN THE PRESSURE AT A GIVEN LEVEL IS CONSTANT AND THE TEMPERATURE AT THIS LEVEL IS A FUNCTION OF THE HORIZONTAL DISTANCE AND A LINEAR FUNCTION OF THE HEIGHT.

Let the gradient ratio which distinguishes one stratification of the air from another having a different temperature gradient be n .

$$(52) \quad P = P_h \left(\frac{T}{T_h} \right)^{\frac{n C_p}{R}}. \quad T = T_h + \frac{g}{n C_p} (h - z).$$

$$(53) \text{ The curves. } F(xz) = n \log T_h - (n-1) \log T = \text{const.}$$

$$(54) \text{ Angle of curves. } \tan \alpha = \frac{\partial F / \partial z}{\partial F / \partial x} = \frac{n}{n-1} \left(\frac{h-z}{T_h} + \frac{C_p}{g} \right) \frac{\partial T_n}{\partial x}.$$

CASE VIII. FINAL CONDITION OF TWO AIR MASSES UNDER CONSTANT PRESSURE WITH GIVEN INITIAL LINEAR VERTICAL TEMPERATURE FALL.

On removing the partition the layers 1 and 2 spread out, change their heights, and there is a mixed stratum between them.

$$(55) \text{ Temperatures } \begin{cases} T_1 = T_{h_1} + \frac{g}{n_1 C_p} (h - z). \\ T_2 = T_{h_2} + \frac{g}{n_2 C_p} (h - z). \end{cases}$$

FIG. 24 E.

$$(56) \text{ Entropy } \begin{cases} S_1 = C_p [n_1 \log T_{h_1} - (n_1 - 1) \log T_1] + \text{const.} \\ S_2 = C_p [n_2 \log T_{h_2} - (n_2 - 1) \log T_2] + \text{const.} \end{cases}$$

$$(57) \quad \log \frac{T_{02}}{T_1} = \log \frac{T_2}{T_{h_1}} = \frac{n}{n-1} \log \frac{T_{h_2}}{T_{h_1}}.$$

$$(58) \text{ Heights } \begin{cases} h_1 = \frac{n C_p}{g} (T_{01} - T_1). \\ h_2 = \frac{n C_p}{g} (T_2 - T_{h_2}). \end{cases}$$

If the vertical temperature fall of the masses 1 and 2 is smaller than in adiabatic equilibrium, then the entropy increases with the height, and it can happen that in the colder

mass (1) the entropy at the height h_1 will be as great as in the warmer mass (2) at the ground. The higher layers in (1) form a series with an entropy equal to the layers in (2) up to the height $h - h_z$. In the final state the under part of (1) will spread out on the ground, above it will be layers which are mixtures of (1) and (2), and farther up will lie the masses of (2) which initially were between $(h - h_z)$ and h . On the boundaries of the three layers the temperature transition is continuous.

It will be convenient to approach the dynamic equations of motion in cyclonic vortices thru a study of the Cottage City

waterspout of August 19, 1896. It should be recognized that in ordinary cyclones the vortices are not perfect and it is only rarely and in highly developed storms that anything like pure vortex motion is attained. The waterspout, therefore, offers a good example of vortex motion in the atmosphere with which to test the above equations. I may remark that the theory first advanced in my International Cloud Report, 1898, for the generation of cyclones and anticyclones in the general circulation seems to be practically confirmed by these studies based upon actual observations.

STUDIES ON THE THERMODYNAMICS OF THE ATMOSPHERE.

By Prof. FRANK H. BIGELOW.

VI.—THE WATERSPOUT SEEN OFF COTTAGE CITY, MASS., IN VINEYARD SOUND, ON AUGUST 19, 1896.¹

THE SOURCES OF THE DATA USED IN THE DISCUSSION.

This waterspout has an especial scientific interest for meteorologists because it was seen under circumstances remarkably advantageous for making observations and photographs, from which it is possible to compute, with much accuracy, the dimensions of the tube, and thus facilitate the application of the mathematical theory of vortices.

A series of papers and letters from various persons who saw the phenomenon, and a very complete set of photographs, were secured at the time by the Editor, which he has courteously placed at my disposal for incorporation in this paper, and they will be found inserted in the following pages. I have myself been familiar with that part of the Massachusetts coast, and have therefore been interested to study the facts as thoroughly as possible, as a preliminary to the discussion of this type of vortex motion. I accordingly visited Cottage City the following September, and was conducted by Mr. Chamberlain to the spot where he placed his camera for making his photographs. There I made a sufficiently accurate survey of the linear distances between that spot and the telegraph poles shown in his pictures to determine the scale of distances for all objects. Furthermore, by collecting and collating all the data relative to the positions of the waterspout and the schooner seen in the several photographs, I am able to plot them on the Coast and Geodetic Survey Chart No. 112, in such a way as to reconcile nearly all of the statements made regarding the distances and progress of the two objects, respectively. The photographs taken from such distances as Vineyard Haven and Falmouth Heights give an excellent view of the whole cumulonimbus cloud from which the tube descended, and its connection with the thunderstorm which preceded it. All these data will enable us to discuss the subject of tornado and waterspout formation with considerable fulness, and with the conviction that confidence may be placed in the comparison of the observations with computations. There is every reason to believe that the photographs are perfectly genuine, and free from touches to add to their artistic beauty at the expense of scientific accuracy. Certain preliminary computations were made in 1897, the result of which was published in the International Cloud Report, page 633, Report of the Chief of the Weather Bureau 1898-99, Volume II; this was republished in the MONTHLY WEATHER REVIEW.² My purpose then was to illustrate the application of certain formulæ, and it was my intention at that time to complete the study as soon as my other duties permitted. In these present papers I shall begin with the descriptive accounts of the waterspout, then pass to a discussion of the facts as shown by these reports and the photographs, and finally consider the dynamic motions and the thermodynamic conditions present in the atmosphere near Cottage City on that occasion.

LETTERS AND REPORTS OF OBSERVERS.

The following letters, reports, and observations have been furnished by the several authors. It will be instructive to refer to fig. 25 while reading these papers.

(A) EXTRACT FROM THE DAILY JOURNAL OF U. S. WEATHER BUREAU STATION, VINEYARD HAVEN, MASS., W. W. NEIFERT, OBSERVER.

August 19, 1896.—Partly cloudy weather during the morning, with gentle northerly wind. Three magnificent waterspouts were observed in Vineyard Sound to-day, in northerly direction from station, about ten miles distant. During the entire afternoon the weather was partly cloudy and sultry, with great masses of cumulus clouds in the north and northeast. At 12:45 the first display was observed. At first a long spiral

column seemed to fall from the clouds, about the thickness of a man's body, but this gradually increased in size as the cloud lowered, and when it reached the water it was as thick as a large sized cask, and changed in color from a rich gray to a black, and assumed a funnel shape at the base of the clouds. The cloud seemed of a yeasty white where the column came in contact with it, and looked as though the water was hauled up to it. The area of contact appeared small. The spout was very straight and almost perpendicular, kicking up a great sea as it traveled. When it disappeared it began to do so at the base and rapidly reached the top, having the appearance of clouds, and finally cleared away, like steam from an engine, at 12:58 p. m., leaving a clear sky for a background and the original clouds above. At 1 p. m. it formed the second time, which was really the most interesting spectacle of all. From a mass of lanky clouds it reached down, finger-like, to almost the ocean's surface. Below it the water was stirred to an angry whirlpool, the foam reaching up perhaps a hundred feet. It appeared as though great volumes of water were traveling up to the cloud by an endless screw, when suddenly, at 1:18 p. m., the long arm disappeared in a manner similar to the first. At 1:30 it formed for a third time and scarcely reached the water, but had a decided funnel shape, lasting about five minutes, when it slowly withdrew into the blackness above and the surface of the ocean became quiet. There was a sprinkle of rain from 12:54 to 1 p. m., amounting to a trace. During the display the wind at the station was six miles per hour from the northwest; temperature 72°, with a fall to 56.5° during the thunderstorm which followed, passing over the station from northwest to south. Thunder was first heard at 1:45 p. m.; loudest at 3:04 p. m.; last at 3:45 p. m. Heavy downpour of rain from 3:04 to 3:15 p. m., then continued light rain until 3:30 p. m. Amount, 0.38 inch. The summer residents were stricken with fear at the approach of the dark clouds over the sound, and viewed the waterspout with mingled feelings of awe and interest. It was a sight long to be remembered, and when the weather cleared, about 4 p. m., each expressed himself as being most fortunate in having escaped some dreadful calamity. No noise was heard here, but the schooner-yacht *Avalon* of Boston was very near the spout and those on this vessel reported plainly hearing the noise and the wind blowing around the vortex with wonderful rapidity; to them the spout appeared to be one hundred feet in diameter. The three spouts moved gracefully to the eastward. This is the first display of this phenomenon witnessed here for 27 years. Mariners here who have circled the globe a number of times, and have seen dozens of waterspouts, declare it to be the most perfect specimen they ever observed.

(B) LETTER FROM MR. NEIFERT TO MR. A. J. HENRY, DATED VINEYARD HAVEN, MASS., DECEMBER 19, 1896.

When I first saw the waterspout it was in the vicinity of Black Buoy No. 13, on the east end of L'Honniedieu Shoal. Can not say now exactly, but in that general direction. Could just see base off East Chop. When the photographic view was taken here it was about where the red dots surround sounding marked 8½. It appeared nearer then, but I presume this was caused by its base being hidden by the "highlands". The view from here was taken from on board of a yacht which lay at the red dot between the two wharfs or the head of the harbor under the figure 3 of the sounding marked 13.³ It may not be so far, but that is as I remembered it. There was so much confusion, women and children crying, that I was not very observant until it was over.

Coolidge was just north of the head of the wharf in Cottage City, and the "spout" was in an east-northeast direction from him. His views were made from the same position, and only time enough elapsed between them to change the plates.

(C) EXTRACT FROM THE DAILY JOURNAL OF THE U. S. WEATHER BUREAU STATION, NANTUCKET, MASS., MAX WAGNER, OBSERVER.

August 19, 1896.—Clear weather all day, except in the afternoon, when light rain began at 2:40 p. m. and ended at 4 p. m. Total amount 0.03 inch. Cooler, with rising barometer. Mr. Wagner went to Cottage City in the morning to check up; from there he observed the big waterspout that formed in Nantucket Sound. An ordinary thundershower was passing across the sound when, about 12:40 p. m., a huge black tongue shot down from an alto-cumulus cloud that floated a half mile high at the northern edge of the shower, and after rising and falling a number of times, finally joined a shorter tongue that seemed to leap out of the water to meet it. Twice the column parted for a moment, but joined again instantly. There was no apparent motion of the waterspout forward, and the phenomenon lasted for half an hour. It was pronounced by many sea captains who witnessed it the finest waterspout they had ever seen. No damage was done by the spout, but a small catboat which arrived at night reported being becalmed near the spout, the crew being badly scared.

(D) EXTRACT FROM THE DAILY JOURNAL, U. S. WEATHER BUREAU STATION, WOODS HOLE, MASS., J. D. BLAGDEN, OBSERVER.

August 19, 1896.—Three waterspouts were reported in the Vineyard

¹ No. V of the series ("The Horizontal Convection in Cyclones") will follow later.

² May, 1902. Vol. XXX, pp. 257, 258.

³ On chart not reproduced.—EDITOR.

FIG. 25.—Location of waterspout seen in Vineyard Sound, August 19, 1896. (Reduced from United States Coast and Geodetic Survey chart No. 112.)

FIG. 26.—Diagram of the survey between site of Chamberlain's camera and four telegraph poles shown in his photograph, 2d A (fig. 27).

Sound and one in Buzzards Bay between 12:35 and 2 p. m. One waterspout was photographed with excellent results.

(E) COPY OF A CIRCULAR ACCOMPANYING COPYRIGHT PHOTOGRAPHS. BY J. N. CHAMBERLAIN, OF COTTAGE CITY, MASS.

About 12:45 noon, August 19, 1896, we were startled by the cry of "A waterspout!" and with our assistants started with the camera to the park in front of Doctor Tucker's residence, where we could see, a little north of the direction of Nantucket, very dark and angry clouds, out of which a funnel-shaped cloud of various colors, with a pointed streak, issued downward until it touched the water. We obtained two photographs of this, showing a slight difference. [One of these views is reproduced as fig. 27.] After about twelve minutes it gradually and completely vanished. Very soon a second one appeared, more curved than the first, with a long sharp streak from the same clouds and slowly extending downwards to a point about one hundred feet from the surface of the ocean. In a few moments this changed to a smaller streak with a different curve bending to the south, while the former bent to the north. Both of these we photographed [figs. 34, 35]. The height of this, which Professor Dwight of Vassar College says was a genuine waterspout, was about a mile. The cloud-burst disturbed the water in the sound for several hundred yards until it looked like a boiling pool. We could trace through the camera the spiral motion of the water as it was drawn into the clouds, every moment augmenting their portentous darkness. The cloud from above and the spray from below were drawn together by suction, and condensed torrents of water poured down a few hours later, which was found by persons in different places on the island to be salt, and proves that it was carried up to a height and scattered round as solid bodies are by tornadoes on land. The Greeks applied the term "prester" to the waterspout, which signifies a fiery fluid, its appearance

being generally accompanied with flashes of lightning and a sulphurous smell showing the activity of the electrical principle in the air.

(F) A REPORT TO THE EDITOR BY PROF. WM. B. DWIGHT, VASSAR COLLEGE, Poughkeepsie, N. Y. DATED MARCH 22, 1897.

I now inclose such statements as I am able to make without the few memoranda, noted on the spot and since lost, of the waterspout of last summer at Cottage City.

The basis of my estimates of the height of the waterspout is rather hypothetical, but I submit them for what they may be worth. I have endeavored to assume my units of measurement so as to be *below* rather than *above* the fact, in order that the estimate might not seem to be made in a spirit of exaggeration. Thus, I am inclined to think that the distance of the schooner, in the photographs, from the shore is nearer three than four miles, which would make the spout higher than my estimate. One reason for my thinking so is that there is a buoy, the three-mile buoy, so called, not far from the position of the schooner and in front of her, about three miles from the shore and marking the channel. She was likely aiming for that buoy and then would not be very far from it. On the other hand, the state of the wind might lead her to go as much as half a mile or more outside (to the east) of it. I presume that the opinion of the seamen at Cottage City on the distance of the schooner could be easily obtained and would be of value. I think that I obtained such an opinion, but it is lost with my other memoranda and I can not now recall it. I have searched for the three-mile buoy in the photographs, but it is a very small object and I cannot identify it.

Some statements as to the waterspout in Nantucket Sound (sometimes called Vineyard Sound) easterly from Cottage City, Murthos Vineyard, Mass., at noon of August 19, 1896, made from personal observations by William B. Dwight, of Vassar College, Poughkeepsie, N. Y. (resident in the summer at Cottage City).

My summer cottage is situated close to the beach at Cottage City, with unobstructed view of the ocean. I was standing upon my private wharf, nearly in front of the cottage, when the waterspout of August 19, 1896, began; I saw it at the outset and was among the first to call general attention to it in our part of the town. I watched it closely, with the assistance of a good field-glass till the close of the phenomenon, but I had no proper instrument at my command for taking the altitude.

Excellent photographs were taken by Mr. Coolidge and Mr. Chamberlain. I am able to testify to their general correctness as corresponding with personal observation. Mr. Coolidge's are most artistic views of the whole scene and scenery. Three of Mr. Chamberlain's present with accuracy three consecutive views of the waterspout in its phases, changes, and progress taken from one and the same spot. They were taken with total disregard of the foreground and the sole aim of getting the best views of the spout itself. These facts give these three views a marked scientific value, and these photographs will repay a careful scientific study.

Like all of the three phenomena of this kind which I have personally observed (and this is the second which I have seen from Cottage City), the funnel of the tornado is constantly changing its form, length, and other dimensions; and occasionally, or at intervals, it may entirely disappear in its cloud, only to reappear again in full force. This one had several such successive appearances with intervals of total disappearance. Hence the photographer, the newspapers, and the spectators generally described the appearance of several waterspouts on this occasion. I consider this an unscientific and unfortunate mode of describing this phenomenon, chiefly for these two reasons.

1. There was only one great but entirely distinct and individual cloud concerned in the phenomenon from beginning to end, and in fact only one particular spot in that cloud. This not only follows from my own observation but is demonstrable from a study of Chamberlain's three photographs, as I propose later to show. This cloud and its point of vortex movement sustained constantly throughout the waterspout phenomenon, three quarters of an hour (or more), the same relation to the furious squall of lightning, thunder, rain, and hail, going on about a mile to the southeast of the waterspout, i. e., a mile from the thunderstorm to the edge of the waterspout. This squall is clearly visible in Chamberlain's first photograph of the three mentioned.

2. From the point mentioned in the tornado cloud, (as I will designate it in distinction from the squall cloud), a waterspout funnel would descend to the ocean, and move along its surface in an easterly direction, with its cloud; after a while it would thin out, or break into pieces, and nearly or quite disappear. For the most part, however, the location of its minimized force in the cloud remained marked clearly by a downward bulging of that part of the cloud, with indications often of rotary movement at the spot. Once, however, the spot where this tendency to vortex motion still existed was for a few minutes lost to view; but soon the vortex movement visibly returned somewhere along the line between the cloud and the ocean, from the point of the cloud which was affected. It generally appeared first at the cloud, but once the vortex movement at the ocean's surface was practically simultaneous with that at the cloud; then another column or spout was completely formed, but as the cloud had been moving eastward during the interval, the spout would of course be seen in a position somewhat to the eastward of its former place; and so this disappearance and reappearance was several times

repeated. Those of the more intelligent observers who insist that there were "several waterspouts" on this occasion base their statement on these two arguments: (1), that there were successive spouts seen; (2), that no two of the spouts were in the same place.

On the contrary, I hold that my preceding remarks, and the further facts to which I shall call attention later, show that this was the same phenomenon, that is, the same center of vortex action, throughout, and that its different appearances were not different waterspouts, but simply different and varying phases of one and the same phenomenon. As to the second point, the difference in position, I contend that the differences in position were only those which a waterspout drawing itself up into its cloud, and then coming down again, must necessarily take in consequence of the constant southeastward progress of the storm. It could not come down in the same place any more than a circus rider can when he leaps up from the back of a running horse and comes down again several feet ahead of his former position. The successive phases of this waterspout, in their positions, follow strictly the eastward movement of the tornado cloud, and inspection of the three photographs of Chamberlain's set shows that a line between the first and last phase of his three would pass through the position of the intervening one.

This may seem a matter of little consequence in terminology; but it is of importance in view of the fact that the expressions "two waterspouts", "several waterspouts", etc., are positively needed for cases where two or more entirely independent phenomena of the kind are in sight at the same time, or nearly so; as when a friend of mine once saw eleven waterspouts on the ocean simultaneously.

I will now give a brief description of the successive phases of the waterspout as I observed it.

I was standing on my own private boat wharf, which is on the seashore at the extreme southeast point of Cottage City, one-half mile exactly south of the "Oak Bluffs" or main wharf, the wharf shown in Coolidge's photograph, No. 7933, fig. 28, a little after half-past twelve. In the excitement of the occurrence I failed to note the exact time. An exclamation from a friend standing near me drew my attention to the waterspout, which had just formed in the rear of a black thundersquall which we had been watching to the southeast, the wind being from the northwest. The waterspout being a mile or more in the rear of the squall and separated from it by a clear interval, was a little north of east from my position of observation; it appeared to be somewhat nearer than the *Suconesset* light-ship (on *Suconesset Shoals*), which is nine (9) miles easterly from Cottage City; at the same time it was evidently nearly as far. I had several interviews subsequently with captains of the local fishing catboats, all men of lifelong experience as coasters, with reference to the probable distance of the spout. They all estimated it as from eight to ten miles away; no one gave a less estimate than eight miles. All but one of the captains had seen it only from Cottage City. One captain, however, told me that he was sailing to Cottage City from Cape Poge, a point seven miles to the southeast of Cottage City, and saw the waterspout when he was off that cape, and that it was certainly nearer to Cottage City than the *Suconesset* light-ship (which from his position would be much in the same direction); he said it was, in his judgment, about one mile nearer to Cottage City than the light-ship; this is excellent testimony on this point, and I think we may safely set the distance of the waterspout from the Cottage City wharf as having been just about eight miles.

At this first phase the waterspout was very tall and very thin; in fact it presented very much the same appearance as in No. 3 of Chamberlain's set, fig. 35, though in a much more northwesterly position; at its base was a spherical mound of up-whirling water and spray several times wider than the main portion of the column, a white dot of foaming water appearing at the center of this mound at the ocean's surface; the column was sinuous and moderately expanded as it joined the cloud. The tornado cloud had a broad, flat, angry looking under surface, little tufts of mist or rain descending from it here and there; it extended at least a mile to the east and southeast, joining the thundersquall in the latter course. Toward the north and west it was much less extensive, and in fact more than one-half of the sky over Cottage City was in bright sunlight. At times it appeared as if streaks of rain were descending from the tornado cloud to the ocean all around the waterspout in all its successive phases.

This first phase is not shown in any of the professional photographs, though probably some amateur's camera may have caught it. Comparatively few persons saw it, that is only those who happened to be at the beach; the morning bathing hour was mostly over; the professional photographers were in their offices inland; it took time to get word to them and for them to bring out their instruments and get them placed in good positions. Meanwhile, this first phase faded away, and that one of the views of the photographers which is generally called the "first waterspout" is not at all the first, but the second phase, and a much larger and grander one. The second phase, which appeared at about a quarter to one o'clock, was by far the grandest one of all. It is the "first" one of the photographers, the one shown by Mr. Coolidge's photograph numbered 7933, fig. 28. It began by the formation of a broad funnel on the under side of the tornado cloud, which then became a very broad black tube. This rapidly stretched down to the ocean, where it raised a large mound of whirling, foaming, rising water at the center, and of

spray around its margin. The white center of upward rushing water was usually clearly visible to the naked eye, and through a field glass was very marked. At other times it was completely obscured by the surrounding mist and spray, and was never relatively large to the view, because so thoroughly enveloped. * * * There was no white water visible at any time in the tube proper, above the mound. This phase lasted probably about fifteen minutes, during which it varied in form from a slender, even cylinder, to a massive, imposing conical tube, as it swept on slowly and majestically to the southeast.

From this phase we are enabled, through Mr. Chamberlain's valuable set of photographs, to trace the forward progress of the tornado cloud visibly, and the relations of this and the succeeding phase to each other, since these three views were taken by the same camera and lens, and from exactly the same point. (I have established this point, as it is easy for any one to do, on the spot, by taking a position on the west margin of the main portion of the Ocean Park adjoining the Oak Bluffs dock, where the relative positions of the telegraph poles, and their several arms and wires can be made to coincide exactly with their relative positions in the photographs. It will be noted that these positions are exactly the same in the three views, though in the last one the camera was revolved more to the southeast than in the others. This point of observation proves to have been at the center of the convex western edge of the main body of the park, just east of the carriage road which extends in a curve from one point of the Sea View avenue to another point of the same avenue around the west edge of this main portion of the park. It is called Ocean avenue. The point where the camera stood is just east of Ocean avenue, where a straight line running through the east end of Fisk avenue would strike it.)

Now, in examining Chamberlain's first view, fig. 27, where the grandest phase is seen, the waterspout is shown a little north of the two central telegraph posts of the view, while a schooner is seen sailing southeast about three or four miles from the shore, some little distance to the north of the waterspout. Also, a little northerly of the waterspout, about a third of the apparent distance (in the view) between it and the schooner, the masts of a vessel at anchor appear on the horizon. This is apparently the *Suconesset* light-ship, nine miles away. It is certainly about the position of that light-ship, and resembles closely its familiar appearance, as seen from Cottage City. This would seem to locate exactly the direction of this phase of the phenomenon from Cottage City. As my charts of Nantucket Sound (or Vineyard Sound) are at Cottage City, and my notes made on the spot are lost, I can not at this time state the bearing of the light-ship from the point mentioned as the position of Chamberlain's camera; but it can easily be settled by reference to such a chart.

Mr. Coolidge's photograph No. 7934, fig. 32, seems to show a later form of this phase, the column having grown thicker and more evenly conical since Chamberlain's first and earlier view was taken. This appears from the fact that the schooner is now much nearer to the line of direction of the waterspout, in which direction she was sailing faster, apparently, than the spout was moving in the same direction. It is true that Coolidge's standpoint was evidently to the north of Chamberlain's, being near the steamboat dock, and quite near the shore, while Chamberlain's position was about five hundred feet from the shore. But when all allowances for difference in position have been made, there seems to be still quite a margin which can only be explained by the fact that the vessel had actually had time to sail some distance southeasterly.

When this grandest phase disappeared the spout was for a few minutes totally absorbed into the cloud. Then, while I was closely watching it for further developments, a whirling funnel began to bulge down from the tornado cloud, while at the same moment, and before the funnel was more than a mere projecting knob on the cloud, the water on the surface of the sound just beneath began to boil furiously, and to rise up in a whirling mound, indicating a line of vortex motion already established the entire distance between the cloud and the sea. Next, while the upper tube began to extend downward, a central portion of the tube was formed, entirely independent of the upper and lower portions, as clear spaces existed between.

This is finely shown in Chamberlain's second photograph, fig. 34, where the three portions are distinctly seen before their union. Many persons on seeing his three views take the one which shows the waterspout as a very slim tortuous tube, fig. 35, as the first exhibition of a waterspout, of which the one in three portions is but the breaking up phase. That this is not the case is indicated by my own most positive observation of the triple formation of the spout, to which I called the attention of bystanders at the time. But, further, the photographs themselves prove the incorrectness of that idea; for it will be noted that in the view of the spout having the triple structure, fig. 34, the column has just passed from its second phase, where it appeared a little to the north of the pair of central telegraph posts, to a position a little to the south of the more northerly of these posts, while the schooner has passed to a central position between them. In the other view, fig. 35, the column has come near to the more southerly of the posts, while the schooner has passed considerably to the south of both, and has been closely approached by a tug towing three barges, which was only just coming into view in the other photograph.

I need only to remark further, therefore, in this connection, that the

third of Chamberlain's set of photographs, fig. 35, represents the completed form of the waterspout shown in the second view, and the third and last actual phase of the entire phenomenon. As this phase disappeared at about 1:25 p. m., the entire occurrence covered a little over three-quarters of an hour.

If one were disposed to form some estimate of the rate of progress of the waterspout, from that of the schooner, the following points would deserve consideration:

1. Though the wind was violent in the vicinity of the squall, there was but a moderate wind on the shore and in the vicinity of the schooner. This is not a matter of my recollection alone, for the photographs show it; the sea near the shore is little disturbed, while the schooner carries all sail except topsails, and has no reefs. If we may estimate the probable length of her hull as 75 feet (a moderate estimate), she must have gone somewhat over half a mile during three-quarters of an hour. Her course being somewhat oblique to the line of vision, and veering away from the observer, is really longer than it measures on the photograph. The tide runs with great force, and may have worked against the schooner.

2. The waterspout was certainly twice as far away as the schooner. Supposing, as is very likely from evidence elsewhere offered, that it was just about twice as far, and moving, as appears to have been the case, in the same course, the tornado cloud and the waterspout must have progressed considerably less than twice as fast as the schooner, since the latter, starting from a spot considerably to the north of the spout passed to the south of it.

Toward the close of this phenomenon the eastern half of the sky became quite black with clouds, while the entire western half, where the Cottage City observers were, was brilliant with sunlight, which at this hour glanced easterly beneath the blackness. The chromatic effects were of an indescribably rare and beautiful kind. The surface of the sound for several miles out was lighted up with weird hues of bright blue, green, yellow, and gray, in patches, according to the nature of the variable weedy and sandy bottom, greatly intensified by the solemn, black storm clouds and waterspout overhead. Thousands of spectators, crowding the beach, gazed on the sight with mingled admiration and awe.

I append some estimates of the probable dimensions of the waterspout founded on its apparent distance, as ascertained by investigation, and upon measurements of the photographs, using as a unit the hull of the schooner in view, estimated as 75 feet long and four miles from shore. I am inclined to think that the schooner's distance was nearer three than four miles, and that the estimates should be increased proportionally. By my estimate the waterspout would have an altitude of about half a mile.

The conditions of the photographs of Chamberlain's series might afford another basis for an estimate.

By ascertaining the distance from the spot where the camera stood to the higher one of the central pair of telegraph posts (as the other stands on lower ground), which distance is not far from 500 feet, and the height of the telegraph post, and assuming the probable distance of the waterspout as eight miles, triangles could be constructed from which the height might be calculated, provided that the telegraph posts were not so near to the camera as to be disproportionately magnified. I suppose corrections might be applied for such irregularity if the power of the lens were known.

Rough estimates as to dimensions of the waterspout seen from Cottage City, Mass., August 19, 1896.

(1) Estimates founded on photograph No. 7934, fig. 32, taken by Coolidge, and based on the supposition that the waterspout was eight miles from Cottage City, and that the schooner visible to the north of it is 75 feet long and four miles from Cottage City. In this photograph, the schooner's hull (not including the bowsprit), estimated at 75 feet, measures one-tenth inch. The waterspout being twice as far distant should therefore measure 150 feet for every one-tenth inch of dimension. Some correction should be made, in strict calculation, for the tendency of the camera to magnify near objects more than distant ones; but this is comparatively slight for two objects both of which are distant, and may be disregarded in a rough estimate.

Using this unit of measurement, 1-10 inch on the photograph corresponds to 75 feet at the schooner, or 150 feet at the spout:

	Feet.
Height from surface of ocean to lower edge of cloud (= 1.917 inches).....	2874
Mound of spray at base of the spout:	
Height.....	600
Breadth.....	750
Breadth of the mass of foaming water in this mound.....	150
Tube or funnel proper, above the base mound:	
Breadth (diameter) just above the mound.....	150
Breadth about the middle.....	300
Breadth at extreme top where it joins the cloud....	600

(2) Estimates founded on a photograph by Chamberlain, fig. 27, and called by him the "first" waterspout. It shows the spout of larger size and grander appearance than the later views. These estimates are based on the assumed length of the schooner in sight, and its distance, assisted by known facts as to the position of the ship channel, and on

the supposed distance of the waterspout, judging from reports of captains of fishing boats who got its range. This is really the second and not the first stage or appearance of the spout, as witnessed by myself from its very beginning.

In this photograph the hull of the schooner (75 feet) measures 13-100 of an inch. Its estimated and probable distance is not over four miles, that of the waterspout eight miles. Hence the unit of measurement, 13-100 inch, would cover 150 feet at the waterspout. The dimensions would then be as follows:

	Feet.
Height from ocean to cloud.....	2600
Height of basal mound of spray.....	300
Width (diameter) of basal mound of spray.....	600
Tube proper:	
Width at lowest point.....	150
Width at middle point.....	100
Width at top.....	375

Remarks.—This photograph represents, as I can testify from personal observation, the same spout, or phase of the spout, as Coolidge's No. 7934; but at a different minute of its existence, since the form is considerably different. The spout was changing constantly in length and width within certain limits, but was throughout the largest of the phases. The difference in the apparent length of the schooner's hull from that in Coolidge's photograph is due probably to a stronger lens.

(G) LETTER OF E. B. HANES, TO THE EDITOR. DATED COTTAGE CITY, MASS., OCTOBER 26, 1896.

Your letter of October 20 is at hand. I am sorry to say that I do not know of any scientific observations of the waterspout seen in Vineyard Sound, August 19. I had an excellent view of it throughout its entire duration, a portion of the time through a six-inch astronomical telescope. It occurred August 19, 1896, at about 1 o'clock p. m. It had been a calm summer day, with but few clouds, temperature about 70°, with but little variation before and after the phenomenon. It has been stated that there were two or three waterspouts; this, I think, is hardly correct, as no one saw more than one at the same time, the so-called different ones being different forms or *reformations* of the same spout. Its beginning was, from my point of view at Cottage City, about six miles distant, in a line toward Cotuit; its ending, about eight miles, in a line toward Hyannis. It had a steady progressive movement and was inclined forward in the direction of advance. I estimate its forward movement at about eight miles in the thirty-five minutes it continued. During the time of the waterspout, showers, with lightning, could be seen preceding and following it in its course; about an hour afterward Cottage City was visited by a tempestuous downpour of rain. Through my telescope the column seemed to be surrounded by a dense vapor, which radiated like smoke from its edges, and, condensing, fell in torrents of rain for a distance in either direction about equal to the diameter of the column. At first the edges of the column were quite well defined, later it grew much larger in diameter and more diffuse, its height remaining the same throughout. While I could not penetrate with my telescope the enveloping mist so as to see if there was a solid or tubular mass of water either ascending or descending the inner part of the spout, nor detect a whirling or spiral movement, yet the funnel shape at the top and general appearance indicated that character. Based upon estimates of the most careful observers, its probable size was from 100 to 300 feet in diameter at different periods, and 4000 to 5000 feet high. Where the column joined the sea there was a great churning and splashing of the water, which extended as white mist for 200, or more, feet upward and outward; this was more pronounced toward the last. When the spout finally disappeared it grew slender and broke about midway of its height, the lower portion dropping into the sea and the upper dissipating into the cloud.

(H) EXTRACT FROM THE REPORT OF THE CLIMATE AND CROP SERVICE, NEW ENGLAND SECTION, AUGUST, 1896, BY J. WARREN SMITH, SECTION DIRECTOR.

On August 19 three well-defined and magnificent waterspouts were observed in Vineyard Sound, between the eastern edge of Marthas Vineyard and the mainland, about off Suconesset Shoal.

Mr. W. W. Neffert, the Weather Bureau observer at Vineyard Haven, writes: "During the entire forenoon the weather was partly cloudy and sultry, with great masses of cumulus clouds in the north and northeast". [The remainder of this quotation is practically identical with (A) above.—EDITOR.]

We have reports of the phenomenon from Mr. E. H. Garrett, who observed it from the coast between Hyannis and Oysterville; from John B. Garrett, who saw it from Falmouth Heights, and from Dr. S. W. Abbott, Secretary of the State Board of Health, who was in West Falmouth Harbor at the time.

Mr. E. H. Garrett says: "We were out on the beach and saw an odd looking cloud in the sky. It seemed to have a curious appendage at first, which one of the party described as looking like 'an icicle.' We turned to go home, when one of the group looking back saw the 'icicle' changing, and we all watched. It grew larger, then looked like a long, thin, gray veil of mist and as it descended the water from the Sound began to rise. I watched it carefully and should say it was over 300 and nearer 500 feet high, and in comparison with the measurement of schooners

lying near it, it certainly could not have been less than 125 feet in diameter."

Mr. John B. Garrett saw it from Falmouth Heights in an east-south-east direction, and its distance was estimated to be from six to ten miles. He says: "In form it was much like a short section of rubber pipe, flexible, and of the color of a heavy watery cloud. It was telescopic, the upper end of the column vanishing in the small end of a funnel-shaped cloud somewhat larger than itself."

"Shortly before it broke and disappeared, the main column drew upward, disclosing at its lower end a smaller column or tube within the main one. There was also visible for a time, as it broke, a distinct spiral and rotary motion, extending about one-third the length of the column from its upper end."

"During the whole appearance the water at its base, considerably wider than the column, was churned into a seething mass and raised to a great height."

"If the estimate of the distance from Falmouth Heights be approximately correct, your previous correspondent's estimate of the diameter of the waterspout, 125 feet, must be within rather than beyond the actual; and, assuming this as correct, the height of the column can not have been less than 750 feet. The height to which the spray was thrown was decidedly greater than the width of the column, and must, therefore, be estimated above 125 feet."

Doctor Abbott estimates the height of the waterspout to have been 3000 or 4000 feet, judging from its appearance above a distant hill, and the "probable distance away of the phenomenon". He says: "From all that I can learn, the waterspout was about 25 miles distant". But it could not have been that distance away from him, and yet have been seen in an "east-southeast" direction from Falmouth Heights, and in a "northerly" direction from Vineyard Haven. Still from his point of view, at ten miles distance, it must have been over 1000 feet in height. He writes that "the waterspout was soon followed by marked atmospheric disturbances. Thunder, lightning, hail, and rain in abundance fell within an hour or more. A dense, dark cloud formed in the northwest, followed by a squall from the southwest, and the wind shifted in a short time from northeast to southeast, and then by southwest to northwest. The thermometer at 2:00 p. m. indicated 56°, a very low reading for a place where it has varied but little from 70° all summer."

(I) COPY OF LETTER OF REV. CRANDALL J. NORTH, OF NEW HAVEN, CONN., IN THE CHRISTIAN ADVOCATE FOR SEPTEMBER 24, 1896.

Thousands of summer residents of Marthas Vineyard, Nantucket, and the adjacent Massachusetts coast were treated to a spectacle of remarkable grandeur one day in August, last. Guests at the hotels and occupants of cottages at the various resorts were just rising from dinner when the cry was raised, "A waterspout, a waterspout!" The scene presented to view was such as not one in a thousand had ever witnessed before or would ever see again.

A large mass of heavy black cloud hung high above the ocean between Nantucket and Cape Cod. Suddenly it was seen to project a circular column of its own dense vapor perpendicularly downward, rapidly but not precipitantly, until sea and cloud were connected by a cylinder one or two hundred feet in diameter, straight as a pine tree, and at least a mile high. It was a waterspout indeed, of most unusual proportions and indescribable beauty.

The sea was perfectly calm, the air almost motionless, the sun shining brightly, light summer clouds hanging here and there over the deep blue sky; and in strange contrast with all the rest, was this lofty mass of black vapor with its absolutely perpendicular support. To add to the weird effect occasional livid streaks of forked lightning shot athwart the black monster cloud above. The column was only slightly funnel-shaped just where it joined the cloud, and was of equal diameter the remainder of its length. At its base the sea was lashed into a mass of white foam and spray that mounted upward as high as the masts of a large schooner.

From Cottage City it seemed about six miles distant, but careful observation through a glass from the writer's view-point showed that it was nearly in line with the light-ship off Hyannis Harbor, and still farther distant, its foot resting upon the sea beyond the horizon line. It must have been twenty or twenty-five miles away, but such was its magnitude that it seemed not more than one quarter of that distance.

It moved slowly eastward, and continued with little change in form for seventeen minutes. Then it gradually attenuated till it looked like a dark ribbon hanging out of the cloud, and at length disappeared. The lashing of the water into foam and spray where its base had rested continued unabated, which was evidence that the waterspout was still there, though now invisible, and that it might be expected to reappear. Surely enough, after an interval of about ten minutes, the cylindrical form of black vapor began to push its way downward again from the cloud and continued until it stood again upon the white mass of foam and spray mounting up from the sea surface. This time its top was more funnel-shaped and curved to the eastward. It continued eight minutes and disappeared. The projecting of the visible vapor downward caused the illusion that its origin was from the cloud rather than from the sea, and many supposed that it was a cloud-burst rather than a waterspout; but this is disproved by the continuance of the agitation of the sea surface

during the interval between the disappearance of the first column of visible vapor and the formation of the second. Also the descent of the column was too slow for a mass of water falling from a cloud-burst, as was clearly apparent a little later, when a real cloud-burst occurred upon the mainland opposite, in full view from our point of observation.

The apparent formation from top downward was due to the fact that the atmosphere became more rarefied by the swifter gyrations of the whirlwind at the higher altitude, causing the invisible vapor carried up from the sea surface to condense and become visible at the highest level first; then its visibility gradually extended downward as the velocity of the gyrations below increased. The whirlwind lashed the sea into foam and spray and vapor, and stood it up in an invisible column; but it turned into cloud at the top first, then downward its entire length, until there it stood for many minutes before the wondering gaze of thousands, a veritable "pillar of cloud by day".

The old sea captains of Marthas Vineyard said that this waterspout exceeded in size and grandeur anything of the kind they had seen during all of their seafaring experience. Enterprising photographers secured several good photographs of the remarkable phenomenon.

(J) COPY OF DESCRIPTION BY DR. F. C. V. H. VOM SAAL; APPARENTLY COMPILED FROM OBSERVATIONS AT COTTAGE CITY, AND PUBLISHED IN THE SCIENTIFIC AMERICAN, NEW YORK, SEPTEMBER 26, 1896.

About 12:30 p. m., August 19, 1896, one of the very dark clouds hovering over Vineyard Sound, between the mainland and Cottage City, was seen to send out a downward and sharply pointed streak of cloud matter, whose funnel-shaped basis above was not at all times visible. After a duration of about fifteen minutes it broke and completely vanished. The apparition quickly emptied of their summer residents all the cottages along the Sound and adjacent islands, Nantucket included. No photographs were taken of this first spout, to my knowledge.

Shortly afterward a long tongue emanated from the same clouds, and was slowly pushed downward to a point about 100 feet from the surface of the ocean. Its height was certainly a mile, and the band-like shape gradually increased in width. With a glass, slow gyratory movements could be detected, also longitudinal stripes caused by falling water. This cloud-burst made the water below, over a surface of many hundred yards, look like a boiling pool. The jumping spray from this was also caught and drawn upward into the whirl toward the downpouring column. This latter, now of lighter color, being struck by the sun, was gradually withdrawn upward, evidently thinning and broadening toward its base. With a glass, mists could still be seen falling into the snow-white foaming area below. The duration of this second and most perfect phenomenon of the day—there were three in all—was about half an hour.

About twenty minutes after its disappearance a third began to form, gradually coming downward from the same clouds, though from a spot a little farther north; but it hardly reached completion. It is very important to note that, in this third case, the ocean below was entirely quiet for a time, being only disturbed later on, when the same process of condensation, mentioned above, caused a similar downpouring, especially noticeable in the period of retraction. It was soon apparent that the agency causing the spouts had spent its energy; the column was evidently thinner in substance and its formation slower and hesitating. It stopped midway, sending only an attenuated end farther, to be withdrawn upward soon after.

During almost all of the time since the appearance of the first spout there was a heavy rainstorm accompanied by flashes of lightning from the northern and darkest portion of the long motionless stratum of clouds above mentioned.

Cottage City, which had been in sunshine until then, was visited by a drenching rain some hours later.

The long duration of the phenomena just described enabled the writer to form a somewhat different opinion of the nature of such waterspouts from what is commonly held. True, I must fall back upon the old (or rather older) explanation, that such whirls are caused by two winds striking each other at an obtuse angle. The greatest rotary velocity must be placed at the spot, about 100 feet above the ocean, toward which the cloud matter from above and the spray from below were drawn. As condensation was continually transforming this cloud matter into water, it stands to reason that by far greater quantities of it were drawn down than was apparent to the eye.

But the spout is from above and not from below, as a glance at the cut conclusively proves. This also definitely settles the question as to what part the ocean takes in the constitution of the column, which is practically none. The "boiling as if in a caldron" is not caused by the action of the circling wind, but by the great quantities of falling water. Nor is there a whirlpool action in, nor rising from, the body proper of the ocean. The way the spray, caught and drawn up, looked at times, easily explained to me how this delusion originated.

The surprising tranquility of the clouds shows that such currents of wind need not be of great height, at least not at their borders, where alone such whirls can take place. That the spouts scarcely shifted their position is proof that the velocity of the concurrent winds was almost

"Of course this was not the "cloud-burst" of technical meteorology, for that is simply an unusual excessive rainfall.—EDITOR.

equal. It is certain that this velocity can not have been great. Several small vessels in close proximity at the time report that there were a great noise and gusts of wind in the immediate vicinity of the display, while beyond this there was almost a dead calm (Boston Globe, September 1). This latter statement, however, seems to be somewhat exaggerated.

THE PHOTOGRAPHS.

We have to acknowledge our debt to the photographers who happened to be in the neighborhood of Cottage City on August 19, 1896, for an admirable series of pictures which cover the most important features of the phenomenon. Messrs. Baldwin Coolidge, 146 Tremont street, Boston, Mass.; J. N. Chamberlain, Cottage City; F. W. Ward, 16 Adams street, Burlington, Vt.; Dodge, of Bangor, Me.; and E. K. Hallet, through Mr. Coolidge have placed their photographs in the care of the Weather Bureau for study, and our thanks are hereby extended to these gentlemen for their courteous contributions to the available scientific data that have come into our possession. Both Coolidge and Chamberlain were stationed on the bluff at Cottage City, and had a clear view over the ocean to the waterspout; Mr. Ward stood near the foot of Hope avenue, in Falmouth Heights; Mr. Dodge was at the head of Vineyard Haven Harbor and saw the spout across the headland near East Chop light; Mr. Hallet was on the high ground west of Cottage City. These locations are shown on the chart, fig. 25. Some other photographs were taken on a small scale which contributed somewhat to the information contained in those reproduced in this memoir.

The first appearance of a waterspout began at 12:45 p. m. and ended at 12:58 p. m.; no photographs were made of this phenomenon, as it required time to bring the cameras into operation. Mr. Coolidge was half a mile away from his studio, at home for dinner, when this spout appeared; he started to secure his instrument, when unfortunately the spout disappeared. He was ready on the bluff for the second appearance, which began at 1:00 p. m. and ended at 1:18 p. m. He used a rapid symmetrical Ross lens, with a focal length $14\frac{5}{8}$ inches from the diaphragm to the ground glass, or $13\frac{1}{2}$ inches from the back of the lens to the ground glass. Mr. Chamberlain brought his camera from his studio to the edge of the park, about 200 yards from the water, and his pictures, therefore, include a foreground showing several telegraph poles. The measured distances between these objects give the scale of the photograph, which becomes more valuable on this account. He used a large camera, No. 5 euroscope, with equivalent focus of $17\frac{1}{2}$ inches from the optical center to the sensitive plate, and a lens of $3\frac{1}{2}$ inches diameter. These sets of photographs by Coolidge and Chamberlain both show a schooner were sailing southeastward, and the positions of the schooner relative to the waterspout in its successive positions are very useful in determining the time intervals between the successive pictures. One of Chamberlain's, fig. 27, also shows the Succoneset Shoal light-ship, together with the waterspout near it, and this is important in identifying the direction of the sight lines from Cottage City. Mr. Ward's picture was taken with an Anthony kodak triad camera, $4\frac{1}{2}$ inches focal length, and the plate is 4 by 5 inches; this was enlarged by Coolidge to the 8 by 10 size. It shows the cloud formation and is most instructive as to general meteorological conditions; it also shows the curvature of the vortex tube at right angles to the view from Cottage City, where it seemed nearly straight, as seen in perspective during the second appearance. Mr. Dodge caught a distant view of the spout, and his picture also shows the great cumulo-nimbus cloud from which it descended; his sight line passed just to the south of the East Chop light-house, and this distinctly identifies the direction of the spout at that time. Hallet's picture was taken with a small camera, but shows the large cumulo-nimbus cloud so well that I have taken it as the basis of the thermodynamic computations.

The following is a list of the photographs and the times when they were taken:

SECOND APPEARANCE.

Fig. No.	Serial No. of photograph.	Phase.	Photographer.	Moment of exposure.	Photographer's numeration of negative.
27	1	2d A	Chamberlain.....	1:02 p. m.	
28	2	2d B	Coolidge.....	1:03 p. m.	7933
29	3	2d C	Hallet.....	1:08 p. m.	
30	4	2d D	Dodge.....	1:12 p. m.	
31	5	2d E	Ward.....	1:14 p. m.	
32	6	2d F	Coolidge.....	1:15 p. m.	7934
33	7	2d G	Coolidge.....	1:17 p. m.	7936

THIRD APPEARANCE.

34	8	3d A	Chamberlain.....	1:20 p. m.	
35	9	3d B	Chamberlain.....	1:24 p. m.	
36	10	3d C	Coolidge.....	1:27 p. m.	7935

Notes on the photographs.

No. 1. See fig. 27.—*Chamberlain, 2d A*, at 1:02 p. m., showing the waterspout 5.75 miles away, the lower face of the cloud in great detail, the foreground, the schooner about two miles out, and Succoneset Shoal light-ship about eight miles distant; the latter can be seen on the horizon about one third of the apparent distance from the spout to the schooner.

No. 2. See fig. 28.—*Coolidge, 2d B*, at 1:03 p. m., includes the Marthas Vineyard steamer, the spout and cloud in nearly the same condition as shown by 2d A.

No. 3. See fig. 29.—*Hallet-Coolidge, 2d C*, at 1:08 p. m. This picture is attributed to E. K. Hallet, photographer, and is copyrighted by Baldwin Coolidge, Boston, Mass., 1897. It seems to be somewhat later than 2d B because the vortex is leaning more toward the south, in accordance with the drift of the cloud stratum, which is brought out more positively in the third appearance, 1:20 to 1:25 p. m.; it also gives us the dimension of the upper cloud which is not seen in the pictures 2d A, 2d B. Such small-scale photographs of the whole cloud region serve admirably to supplement the details to be found only on the large-scale pictures, and should always be made if possible by those having kodaks at hand.

No. 4. See fig. 30.—*Dodge, 2d D*, at 1:12 p. m., is chiefly of importance in locating the line from the head of Vineyard Haven Harbor to the waterspout. The curvature toward the southwest in the center begins to be seen from that angle. I estimate that this was taken at 1:12 p. m., though there may be some doubt about the exact minute.

No. 5. See fig. 31.—*Ward-Coolidge, 2d E*, probably at 1:14 p. m., taken by F. W. Ward, enlarged and copyrighted by Baldwin Coolidge. The curvature of the tube is now fully seen from Falmouth Heights, where this plate was taken, this sight line being nearly at right angles to those from Cottage City. The vortex column appeared vertical at Cottage City, but strongly curved at Falmouth Heights with convexity toward the southwest. In the third appearance the convexity is seen nearly broadside on at Cottage City, and this indicates some change in the drift of the lower surface of the cloud relative to the layer of air at the water. This photograph gives the horizontal extent of the cloud. The Hallet photograph, fig. 29, 2d C, shows the precipitation in the thunderstorm preceding the waterspout by about one mile.

No. 6. See fig. 32.—*Coolidge, 2d F*, at 1:15 p. m., shows the enlargement of the tube before breaking up, the spray being cast out from all parts of the tube, especially at the top, thus causing the conical form.

No. 7. See fig. 33.—*Coolidge, 2d G*, at 1:17 p. m., gives the phenomenon at the breaking up of the second appearance, and it locates the schooner well up to the place of the vortex.

There are three photographs of the third appearance.

No. 8. See fig. 34.—*Chamberlain, 3d A*, at 1:20 p. m., shows the top of the vortex advanced toward the south relative to the base, indicating the drift in the cloud stratum. The schooner has moved beyond the base of the waterspout and is between the two telegraph poles; a tow of barges is just coming into view on the extreme right of the photograph.

No. 9. See fig. 35.—*Chamberlain, 3d B*, at 1:24 p. m., is similar to the preceding, but the base of the spout has moved toward the southeast; the schooner and the barges are approaching each other.

No. 10. See fig. 36.—*Coolidge, 3d C*, at 1:27 p. m., is a later phase of the third appearance, with the schooner and head of the tow nearly in the same line. The schooner is about two and one-half and the barges about three miles distant from Cottage City.

POSITION OF THE WATERSPOUT IN THE SOUND.

It will be seen that from the foregoing notes, the photographs, and the chart we have considerable data with which to find the position of the waterspout in Vineyard Sound. It will be best first to fix our attention upon the first part of the second appearance as shown in the photograph, *Chamberlain, 2d A*, fig. 27, taken at about 1:02 p. m. My own personal survey of the ground gives the following distances approximately, as plotted in fig. 26. The telegraph poles are marked 1, 2, 3, 4, and *Chamberlain's* camera is marked 5. We have the distances, 5-1=450 feet, 5-4=504 feet, 1-2=72 feet, 2-3=120 feet, 3-4=66 feet, 4-1=132 feet. The sight line to the waterspout is laid down, also that to the schooner; the angular distance between them is 5.44° . On photograph, *Chamberlain, 2d A*, fig. 27, is also shown the *Succoneset Shoal* light-ship, which appears as a dot on the horizon about one-third the distance from the foot of the spout to the schooner. This enables us to orient the entire drawing with great accuracy. These lines are now transferred to the chart of Vineyard Sound, published by the U. S. Coast and Geodetic Survey as No. 112, August 1901, of which a portion is reproduced as fig. 25.

On photograph, *Dodge, 2d D*, fig. 30, taken from the head of Vineyard Haven Harbor, the spout is shown just to the south of East Chop light-house, and that line is added to the chart. The spout was also seen from Woods Hole at the head of Little Harbor, and a measurement of the line as described gives magnetic declination $S. 75^\circ E.$, which is also drawn. It was seen from Edgartown, 10° east of true north, by one report, this line being indicated on the chart, fig. 25.

Dr. George Faulkner's family saw the second appearance from their residence near the water in the town of Falmouth, and their sight line passed just south of the steamboat wharf at Falmouth Heights. This enables us to fix another line as shown on the chart. These lines all converge quite accurately to a point a little south of east of L'Hommedieu Shoal, where other observers also placed it by estimate, and I have accordingly cut off the *Chamberlain* line of sight at that point on my chart, fig. 25. This makes the distance from *Chamberlain* to the waterspout 5.75 miles, and to the *Succoneset Shoal* light-ship eight miles; as the schooner was in the usual inside channel it was about two miles distant, as shown on this same chart. It is instructive to note that some spectators imagined the spout to have been more than twenty miles from Cottage City. It is of great importance to be able to accurately convert the distances shown on the photographs into angles, because the angles, combined with the length of the sight line, give the corresponding linear dimensions at the spout and at the schooner. We have to measure the linear distance on this photograph from the middle of the schooner to the middle of the waterspout, which is 48 millimeters on fig. 27, *Chamberlain, 2d A*; at the same time the angular distance between the sight lines from the camera to these two objects is found from the survey to be 5.44° . This was determined by plotting the

lines of the survey on a large scale, and testing the result by numerous checks on the other distances measured on the photographs. Hence, 1 millimeter = $6' 48''$ of angle. This is the fundamental dimension, and it leads to 1 millimeter = 60 feet = 18.3 meters at the waterspout.

DIMENSIONS AS MEASURED ON PHOTOGRAPH 2d A, FIG. 27.

By this process we obtain the absolute dimensions given in the accompanying table.

Distance from middle of schooner to middle of waterspout on the horizon, measured on the photograph, 48 mm.

Angular distance subtended by the sight lines at the camera, as determined by the local survey, 5.44° .

Hence, 1 mm. subtends $5.44^\circ \div 48 = 0.1133^\circ = 6.80' = 6' 48''$.

	Feet.	Meters.
At the distance 500 feet, 1 mm. is equivalent to.....	0.989	0.3014
At the distance of 1 mile, 1 mm. is equivalent to.....	10.444	3.1833
At the distance of the schooner, 1 mm. is equivalent to.....	20.888	6.3667
At the distance of the waterspout, 1 mm. is equivalent to.....	60.00	18.288
Length of schooner hull (3 mm.).....	62.7	19.111
Length of schooner over all (4 mm.).....	83.6	25.481
Diameter of the waterspout at the water (4 mm.).....	240	73.15
Diameter of the foot of the cascade (12 mm.).....	720	219.46
Height of the cascade (7 mm.).....	420	128.02
Diameter of the vortex tube at middle (2.4 mm.).....	144	43.89
Diameter just at face of the cloud (14 mm.).....	840	256.03
Approximate length of the tube (60 mm.).....	3600	1097.3
Approximate height of the top of the cloud (from 2d C, fig. 29)...	18000	4876.8

The distance moved by the waterspout from the beginning of the first appearance at 12:45 p. m. to the end of the third appearance at 1:28 p. m. can be found as follows:

The positions of the schooner and the waterspout at the time of taking *Chamberlain's* three photographs are shown on the chart (see fig. 25), as nearly as can be determined; *2d A*, at 1:02 p. m.; *3d A*, at 1:20 p. m.; *3d B*, at 1:24 p. m. In the interval, 1:02 to 1:24 p. m., 22 minutes, the schooner moved about 0.65 mile. This is at the rate of 1.7 miles per hour. The schooner was sailing nearly east-southeast, and the sails were set to catch a wind from the northwest. The wind was very light at the time, as stated by several observers, and as is shown on the photographs by the smoothness of the water. In the interval, 12:45 to 1:28 p. m., 43 minutes, the vessel passed over the distance 1.27 miles. Similarly, the waterspout passed over the distance 0.4 mile in the interval, 1:02 to 1:24 p. m., and over the distance 0.78 mile, or 4018 feet, in the interval, 12:45 to 1:28 p. m., while the whole phenomenon was in evidence. This is at the rate of 1.10 miles per hour.

It is instructive to compare these results with the estimated dimensions and distances as reported by different spectators. Mr. Hanes estimated the eastward progress as 2 miles, diameter from 100 to 300 feet, height 4000 to 5000 feet. Mr. North made the distance of the waterspout from Cottage City 20 miles, or more, supposing that the foot of the vortex was beyond the horizon, and that from his view-point the base of the tube was 20 feet above the sea level; he made its eastward movement about equal to its own height before it disappeared, which is nearly correct, and called this one mile. Mr. Coolidge, October 19, 1896, estimated the height of the spout at from 6000 to 10,000 feet, or 21 to 28 times its diameter, and the latter at 300 to 375 feet and the distance 8 miles. Mr. Coolidge, September 1, 1897, made it 400 to 600 feet in diameter at its mid-height, from 4000 to 6000 feet, or perhaps 10,000 feet high, and 5 miles distant. The observers on the yacht *Avalon*, which was very near the waterspout, made the diameter 100 feet. Mr. E. H. Garrett estimated over 300 to nearly 500 feet high, and 125 feet in diameter; Mr. John B. Garrett; 6 miles distant, height, 750 feet; diameter, 125 feet; height of cascade, 125 feet; Mr. Abbott; height, 3000 to 4000 feet.

[The treatment of this waterspout will be continued in Sections VII, VIII, and IX.]

FIG. 27.—2d A; second appearance; Chamberlain; Cottage City; 1:02 p. m.

FIG. 28.—2d B; second appearance; Coolidge; Cottage City; 1.03 p. m.

WATER SPOUT IN VINEYARD SOUND, WEDNESDAY AUG. 13, 1895, AS SEEN FROM COTTAGE CITY. COPYRIGHTED BY BALDWIN COLLOIDAL PHOTOGRAPH CO. BOSTON, MASS. 1897.
 E. N. HALL, LITH.

FIG. 29.—24 C; second appearance; Hall's; Cottage City; 1:08 p. m.

FIG. 30.—2d D; second appearance; Dodge; Vineyard Haven; 1:12 p. m.

FIG. 31.—24 E; second appearance; Ward; Falmouth Heights; 1:14 p. m.

FIG. 32.—2d F; second appearance; Coolidge; Cottage City; 1:15 p. m.

WATER SPOT, VINEYARD SOUND, LONG ISLAND, LAT. 41° 34'.
FROM 12:45 TO 1:05 PM - WEDNESDAY, AUGUST 18, 1896.
COPYRIGHTED BY BALDWIN COOLIDGE, BOSTON, MASS.
NO. 7936.
T. J. HARRIS, D.P.

Third phase of second

29

FIG. 33.—2d G; second appearance; Coolidge; Cottage City; 1:17 p. m.

FIG. 34.—3d A; t'bird appearance; Chamberlain; Cottage City; 1:20 p. m.

FIG. 35.—3d B; third appearance; Chamberlain; Cottage City; 1:24 p. m.

WATER SPORT, VINEYARD SOUND, LONG ISLAND, L.A.S. 1901.
FROM 1845 TO 1918. C.M. WEDNESDAY, AUG. 15, 1891.
COPYRIGHTED BY BALDWIN COOLIDGE, BOSTON, MASS.
1891.

Third Sport
All three sports dropped from the same place in the cloud.
3d C. in succession.

FIG. 36. —3d C; third appearance; Coolidge; Cottage City; 1:27 p. m.

STUDIES ON THE THERMODYNAMICS OF THE ATMOSPHERE.

By Prof. FRANK H. BIGELOW.

VII.—THE METEOROLOGICAL CONDITIONS ASSOCIATED WITH THE COTTAGE CITY WATERSPOUT.

The data that have been collected regarding the meteorological conditions prevailing at the time of the Cottage City waterspout are sufficiently extensive and accurate to enable us to study carefully the causes that produced the phenomenon, and to derive several important results regarding the formation of lofty cumulo-nimbus clouds and the dynamic actions going on within them. In this instance we can compute approximately the forces producing the ascension of the buoyant vapor in the cloud, the formation of hail and the energy working in the vortex at the base of the cloud which developed as the waterspout. I shall proceed to give these facts in detail, as this computation may serve as a type to be followed in discussing other cases of similar local atmospheric action.

Besides the formation and dissipation of the tube, noted in the several reports and shown in figures 27–36, there are special features to which attention must be directed: (1) In the third appearance the vortex tube shows a gradual tapering of the form from the cloud to the sea level, but in the second appearance the tube seems to have about the same diameter from the cloud to the sea level. It is necessary to account for this divergence in the type. (2) The photographs show a peculiar set of boundary curves in the cloud level which depend upon certain dynamic forces that we shall attempt to discover. (3) At the foot of the tube, near the sea level, there was a great commotion of the waters, with a white nucleus just under the tube, and finally a beautiful cascade of imposing dimensions surrounding it. These are topics of especial interest besides those usually considered in discussing such vortices.

The meteorological conditions are given quite fully by the regular observations of the neighboring Weather Bureau stations, Nantucket being a station of the first order and having a continuous barograph and thermograph record; Woods Hole, a station of the second order, with complete daily evening observations; and Vineyard Haven, a station of the third order, with daily temperature, wind, and cloud reports. The daily weather map of 8 a. m., August 19, 1896, exhibits the general conditions for the United States, and from it can be obtained the local conditions prevailing at that hour, at least approximately. The physical appearance of the waterspout has been described fully in the reports already given, and there is also a series of notes of which further use will be made in the proper places. We shall endeavor in this Section, VII, to discuss the scientific problems which are naturally suggested by these data, with the view of illustrating typical methods of treating waterspout and tornado phenomena whenever these occur.

METEOROLOGICAL CONDITIONS FOR AUGUST 19, 1896.

Vineyard Haven, Martha's Vineyard, Mass.—The Journal for this station has been given as the report of W. W. Neifert, the observer, in the preceding Section, VI, page 307, extract A.

Nantucket, Mass.—The Journal for this station has been given as the report of Max Wagner, the observer, on page 307, extract C.

Woods Hole, Mass.—The report for this station has been given as the report of J. D. Blagden, the observer, on page 307, extract D.

This last report also adds:

Clear during the forenoon, partly cloudy during the afternoon. Thunderstorm: thunder first heard, 1:58 p. m.; loudest, 3:02 p. m.; last, 3:50 p. m. Storm came from the northwest and moved toward the southeast; temperature before the storm 66°, after 67°; direction of the wind before the storm northwest, after, northwest; during the storm the wind shifted to the northeast. Rain began 2:55 p. m.; ended 3:20 p. m.; amount 0.33 inch. Maximum wind velocity 38 miles per hour from the northwest, at 3:00 p. m. A few hailstones fell about 3:10 p. m., and quite a heavy fall of hail was reported a few miles north of this office.

The weather map of August 19, 1896, is represented as fig. 37.

In Section A of Table 50 the meteorological data are given for Woods Hole, Vineyard Haven, and Nantucket on August 19, 1896. They are extracted from Forms 1001–Met'l. of Woods Hole and Nantucket, and Form 1004–Met'l. of Vineyard Haven. The notation is as follows: *B*=barometric pressure; *t*=dry-bulb thermometer; *t*₁=wet-bulb thermometer; *d*=dew-point; *R. H.*=relative humidity; *e*=vapor tension; *Max. t* and *Min. t*=maximum temperature and minimum temperature for the periods ending at the respective times of observations; *Dir.*=direction and *Vel.*=velocity of the wind; *Amt.*=amount of precipitation; *Amt.*=amount; *Kind*; *Dir.*=direction from which the clouds came; *Local time*=hour of making the regular observations.

In Section B are given the meteorological data at the evening observation for ten days immediately preceding, and ten days immediately following the date August 19, 1896, with the purpose of showing the kind of August weather prevailing in that locality.

In Section C are given data at alternate hours obtained from the continuous self-register of the pressure and the temperature for Nantucket during August 19, 1896.

PROBABLE CONDITIONS NEAR THE WATERSPOUT.

The general chart for August 19, 1896 fig. 37, shows that an area of high pressure was central over the upper Lake region, with its eastern edge just overlapping the southern New England coast. This anticyclone was advancing quite rapidly for the summer season, and on the following day, August 20, it extended far eastward over the ocean. The winds were light to fresh from the north and northwest over New England, and the advancing border of the area of high pressure produced a showery condition with precipitation in eastern Maine, the upper St. Lawrence Valley, and on the Massachusetts coast. Later in the day several thunderstorms developed in the neighborhood of Vineyard Sound, such storms being reported as follows, seventy-fifth meridian time:

Station.	Began.	Loudest.	Ended.	Precipitation.
Woods Hole.....	1:58 p. m.	3:02 p. m.	3:50 p. m.	2:55 to 3:20 p. m.; 0.33 inch.
Vineyard Haven.....	1:45 p. m.	3:04 p. m.	3:45 p. m.	3:04 to 3:30 p. m.; 0.38 inch.
Nantucket*.....				2:40 to 4:00 p. m.; 0.03 inch.

*Note by observer at Nantucket, Mass.—An ordinary thunderstorm was already passing across the sound, when about 12:40 p. m. a huge black tongue shot down from an alto-cumulus cloud that floated half a mile high at the northern edge of the shower.

Several observers mention the thunderstorm that occurred in the neighborhood, in connection with the formation of the waterspout at the base of a large cumulo-nimbus cloud. The day had been generally calm, the breeze being about three to six miles per hour from the northwest. There was a pronounced turbulent congestion of the atmosphere along the southeastern edge of the anticyclone, also a strong convectional movement in the vertical direction, as was indicated by the rapid formation of clouds, the production of precipitation, and the generation of thunderstorms accompanied by an overturning of the strata. All these are consequences attending the flow of cold anticyclonic air over the ocean, causing abnormal temperature stratifications to be superposed upon the ordinary quiet arrangement due to solar insolation taken by itself. The readjustment of the thermal equilibrium, which had become much congested, gave rise to the phenomena

FIG. 37.—Weather conditions, Wednesday, August 19, 1896, at 8 a. m., seventy-fifth meridian time, preceding the waterspout.

observed on the occasion, one of which was the formation of the waterspouts from the base of the same thunderstorm cloud. The vertical convection producing the cloud became vigorous enough to generate vortex tubes during the interval, 12:45 to 1:28 p. m. Similar violent disturbances of the lower atmosphere occur in summer whenever masses of air of very different densities are brought close together, with abrupt changes in the temperatures within short distances. In the Mississippi Valley this usually occurs in the southeast quadrant of a cyclone, where the two streams flow together or overflow one another, one cool and dry from the northwest, the other warm and moist from the south. Near the Atlantic coast the same effect is produced by an anticyclonic area with dry, cool northwest winds pushing forward and protruding the current, flowing freely in the higher levels, over the warm, moist layers of air lying near the surface of the ocean. In all such cases we have a natural thermodynamic engine, where the pressure, volume, temperature (p_1, v_1, T_1) for the "source" of the thermal energy is quite different from (p_0, v_0, T_0) for the "sink", the former corresponding with the boiler and the latter with the condenser of an engine. Thus, (p_1, v_1, T_1) apply to the meteorological conditions in the warm air over the ocean before the anticyclone disturbs it, and (p_0, v_0, T_0) to the conditions prevailing in the cool air of the anticyclone itself. The motions of the atmosphere, which result in clouds, thunderstorms, and waterspouts simply represent the stream lines through which the air flows in restoring the abnormal temperatures to equilibrium. In nature there is a tendency to produce a succession of such thermal engines throughout the atmosphere, first on a large scale due to solar radiation, with the boiler along the Tropics, and the condenser around the

poles; second on a small scale, wherever in the local circulations, which are secondaries derived from the first kind, masses of air of different temperatures come into close juxtaposition. Similar processes continue from step to step, from the general circulation on the entire hemisphere of the earth, to the smallest whirl that occurs in the turbulent internal vortex motions of the air.

Were it possible to trace the course of these stream lines throughout such congested masses as are in motion in tornadoes and thunderstorms, we should be able to study many interesting problems in hydrodynamics as well as in thermodynamics.

Examining the meteorological data I make the following deductions. *The temperature at the base of the waterspout was about 67.5°. This is the maximum temperature of the day, and the thermograph for Nantucket shows that the waterspout occurred at the time of maximum temperature, just before the break in weather occurred. It then fell to about 56.5° at Vineyard Haven, and 59.0° at Woods Hole and Nantucket, so that the effective temperature in the anticyclone was about 58.0°. Hence we shall assume $t_1 = 67.5^\circ$ F. and $t_0 = 58.0^\circ$ F. The pressure in the anticyclone may be taken 30.10, and on the ocean near the waterspout 30.05, which is only a little lower. The table makes the relative humidity range above 90 per cent at Nantucket for several days till August 18. By the morning of August 19 a decided change had occurred, in which the relative humidity fell nearly to 60 per cent. At Nantucket it was 60 per cent at 8:20 a. m., and 74 per cent at 8:20 p. m.; at Woods Hole it was 61 per cent at 8:17 p. m. It is very remarkable that this great waterspout was produced on the day when the lower strata of the atmosphere were drier than on any other*

(A) TABLE 50.—*Meteorological data for August 19, 1896.*

Station.	Pressure.	Temperature and moisture.							Wind.		Precipitation. Amt.	Clouds.			Local time of observa- tion.
	<i>B_o</i>	<i>t</i>	<i>t₁</i>	<i>d</i>	R. H.	<i>e</i>	Max. <i>t</i>	Min. <i>t</i>	Dir.	Vel.		Amt.	Kind.	Dir.	
	<i>Inches.</i>	<i>° F.</i>	<i>° F.</i>	<i>° F.</i>	<i>Per cent.</i>	<i>Inches.</i>	<i>° F.</i>	<i>° F.</i>		<i>Miles p. h.</i>	<i>Inch.</i>				
Woods Hole.....	30.15	64.0	56.2	50.0	61	.360	64.0	59.0	nw.	14	0.33	0	0	0	8:17 a. m. 8:17 p. m.
Vineyard Haven.....							72.0	56.5	nw.		0.38				8:17 p. m.
Nantucket.....	30.03	64.0	56.5	51.0	62	.373	64.0	59.2	n.	10	0.02	few	Cl.	w.	8:20 a. m.
	30.13	62.0	57.0	53.0	74	.402	67.5	61.5	n.	8	0.03	0	0	0	8:20 p. m.

(B) *Records for 10 days either side of August 19, 1896.*

Dates.	Woods Hole, 8:17 p. m.							Nantucket, 8:20 p. m.						
	<i>t</i>	<i>t₁</i>	<i>d</i>	R. H.	<i>e</i>	Max. <i>t</i>	Min. <i>t</i>	<i>t</i>	<i>t₁</i>	<i>d</i>	R. H.	<i>e</i>	Max. <i>t</i>	Min. <i>t</i>
	<i>° F.</i>	<i>° F.</i>	<i>° F.</i>	<i>Per cent.</i>	<i>Inches.</i>	<i>° F.</i>	<i>° F.</i>	<i>° F.</i>	<i>° F.</i>	<i>° F.</i>	<i>Per cent.</i>	<i>Inches.</i>	<i>° F.</i>	<i>° F.</i>
August 9.....	73.0	71.8	72	94	.783	78.2	72.2	70.0	70.5	71	98	.757	78.0	70.2
August 10.....	76.0	71.5	69	80	.707	85.8	73.0	72.5	71.0	70	93	.732	86.3	72.5
August 11.....	76.7	72.5	70	82	.732	87.0	75.0	74.5	72.5	72	91	.783	83.2	74.5
August 12.....	75.2	73.0	72	90	.783	85.1	75.2	71.0	70.0	70	95	.732	81.8	71.0
August 13.....	72.5	71.5	71	95	.757	81.9	72.5	71.0	70.0	70	95	.732	82.0	71.0
August 14.....	66.5	66.0	66	98	.638	71.0	66.5	66.0	66.0	66	100	.638	74.0	66.0
August 15.....	65.2	64.0	63	94	.575	69.0	65.0	64.0	63.5	63	97	.575	67.5	64.0
August 16.....	69.2	67.0	66	89	.638	71.0	65.9	65.0	64.5	64	97	.595	66.8	63.2
August 17.....	68.8	63.8	61	76	.536	77.7	68.5	66.5	64.5	64	90	.595	74.2	66.5
August 18.....	62.2	58.2	55	79	.432	69.8	60.3	64.0	63.5	63	97	.575	73.0	64.0
August 19.....	64.0	56.2	50	61	.360	67.4	64.0	62.0	57.0	53	74	.402	67.5	61.5
August 20.....	64.8	59.0	55	71	.432	70.0	59.1	69.5	55.5	52	78	.387	69.0	69.0
August 21.....	65.0	58.3	54	67	.417	71.0	64.5	57.5	56.5	56	94	.448	68.6	57.5
August 22.....	67.0	61.0	57	71	.465	71.8	65.9	64.5	60.0	57	77	.465	71.0	64.5
August 23.....	71.0	70.2	70	96	.732	73.3	68.2	69.0	68.0	68	95	.684	74.6	66.1
August 24.....	71.6	69.0	68	88	.684	76.1	69.5	67.5	65.5	64	90	.595	75.5	67.5
August 25.....	65.0	62.8	62	89	.555	75.7	64.0	64.5	64.0	64	97	.595	74.8	64.0
August 26.....	68.0	65.0	63	85	.575	73.2	66.3	64.0	62.5	62	92	.555	73.8	63.5
August 27.....	68.0	65.0	63	85	.375	73.8	67.0	63.5	67.0	66	93	.638	74.0	63.3
August 28.....	65.3	58.8	54	68	.417	70.7	60.8	61.5	56.5	62	74	.387	68.0	61.5
August 29.....	64.0	58.0	54	70	.417	68.0	62.9	58.0	56.0	65	89	.432	68.8	58.0

(C) *Continuous self-register of pressure and temperature at Nantucket, Mass., August 19, 1896.*

Hours of seventy-fifth meridian time.	Mid't.	2 a. m.	4 a. m.	6 a. m.	8 a. m.	10 a. m.	Noon.	2 p. m.	4 p. m.	6 p. m.	8 p. m.	10 p. m.	Mid't.
Pressure, in inches.	29.99	29.97	29.96	30.00	30.03	30.05	32.05	30.05	30.07	30.09	30.12	30.15	30.16
Temperature, degrees Fahrenheit.	61.0	60.0	61.0	62.5	64.0	65.0	66.0	67.0	63.0	64.0	63.0	63.5	63.0

day of that month. A comparatively low humidity was prevailing during the formation of the vortex, and even the thunderstorms of the afternoon did not avail to increase it much, as it rose only to 74 per cent in the evening. There was no doubt considerable local fluctuation in the humidity for short intervals, but the prevailing anticyclone lowered the humidity for the duration of two or three days. We have therefore the special problem of accounting for the waterspout during conditions which were quite the reverse of those usually assumed to prevail. Tornadoes are usually found connected with warm, moist air, but here we find cool, dry air, showing that it is not high surface temperature and humidity alone that causes these vortices. After several trial computations on the cloud dimensions, which depend upon the correct surface data, and allowing a small rise in the humidity percentage from 8:17 a. m. to 1 p. m. over the 61 or 62 per cent prevailing at the early hour, I have concluded to accept $R. H. = 64$ per cent as that prevailing near the surface of the water at the time the waterspout was formed. We therefore have to begin the computations with the following data for the air just above the surface of the water at 1 p. m.:

$$B = 30.05 \text{ inches.}$$

$$t = 67.5^{\circ} \text{ F.}$$

$$R. H. = 64 \text{ per cent.}$$

COMPUTATION OF THE PRESSURE B , TEMPERATURE t , VAPOR-TENSION e , AND HEIGHT H , FOR THE α , β , γ , δ STAGES.

We now have the meteorological data at sea level beneath

the cloud which surmounted the waterspout, as shown in the photograph (fig. 29, 2d C), taken at about 1:08 p. m., by Mr. E. K. Hallet. This cloud is a large cumulo-nimbus; its flat base is about 3500 feet above the sea level, and its apex about three miles high. Upon the southern extension, on the right-hand side of the plate, there is a thunderstorm and the rain is falling freely; the northern side is clearing, and there seems to be sunshine in several places; from the midst of the cloud the waterspout projects down to the sea level. At the top of the waterspout tube the curvature of the vortex is well defined, and at the bottom there is a fountain of water surrounding it in a circular cascade. We shall determine as accurately as possible the meteorological elements (B, t, e) at the base of the cloud, that is, at the top of the α -stage, which extends from the sea level to the lower face of the cloud, and expresses the fact that this stratum of air is composed of dry air and invisible aqueous vapor mixed in certain proportions, as indicated by the relative humidity, which is 64 per cent at sea level and 100 per cent at the base of the cloud. We shall use the notation:

h_p, B_p, t_p, e_p at the top of the α -stage, or the base of the cloud.
 h, B, t, e at the bottom of the α -stage, or the sea level.

Compare the notation on page 677 of the International Cloud Report.

(1) The working formula for the α -stage is No. 145, as given on page 496 of the same cloud report:

$$C_a = \text{Constant} = \left(0.2374 + 0.1512 \frac{e}{B} + 0.0232 \frac{e^2}{B^2}\right) \log T. \quad \text{I}_a$$

$$(145) \quad - \left(0.06858 + 0.02592 \frac{e}{B}\right) \log B. \dots \text{II}_a$$

where T is the absolute temperature $= 273^\circ + t^\circ$ centigrade. This formula is reduced to the numerical Tables 94, 95, 96, pages 550 to 553 of that report, inclusive, the T and B terms being summarized by $C_a = \text{I}_a + \text{II}_a$. Some examples of the use of these tables are given in that report on page 573 and following, also page 765 and following, the accompanying text having the necessary precepts for practical work. They will be further illustrated in the examples offered by this waterspout.

(2) The β -stage extends from the base of a cloud to the plane of freezing temperature, $t = 0^\circ\text{C}$., which in this cumulus cloud lies at about 2800 meters, or 9200 feet, above sea level, so that the β -stage stratum is not far from 1700 meters or 5600 feet thick. In the present case the α -stage is about two-thirds of a mile deep, and the β -stage a little more than one mile deep. The β -stage consists of a mixture of dry air, invisible aqueous vapor, and condensed aqueous vapor in the form of minute water drops or cloud particles which reflect and refract the light, and thus make the cloud appear as a visible mass or a region of condensation. The formula for the β -stage is No. 146 as given on page 496:

$$C_\beta = \text{Constant} = \left(0.2374 + 0.4743 \frac{e}{B} + 0.145 \frac{e^2}{B^2}\right) \log T_1. \dots \text{I}_\beta$$

$$(146) \quad - \left(0.06858 - 0.04266 \frac{e}{B}\right) \log (B_1 - e_1). \dots \text{II}_\beta$$

$$+ \left[\frac{497.5}{T_1} - 0.4404 - \left(\frac{309.4}{T_1} - 0.2739 \right) \frac{e}{B} \right] \frac{e_1}{B_1 - e_1}. \dots \text{III}_\beta,$$

where the notation is taken from Table 133, page 677, of the Cloud Report. The numerical tables are given on pages 554 to 556 and Tables 97, 98, and 99 for the successive terms, I_β , II_β , III_β , and examples of the computation may be found on pages 574, 696 and following, also in this paper.

(3) The γ -stage stratum is shallow, in this case only 75 meters or 246 feet thick, and it contains the layer of atmosphere within which the water of the cloud is turning to ice, the temperature remaining constant at the freezing point during this process. In this layer the aqueous content is changing its physical state, though not its temperature, and there is only a mixture of aqueous vapor, water, and ice, since there is no thermal change in the dry air because of the constant temperature, the cooling by expansion just balancing the warming due to evolution of latent heat.

This is a very interesting process and will be referred to again in a later section of this paper. The formula is No. 147, on page 496, in the notation of the Cloud Report, page 677:

$$C_\gamma = \text{Constant} = - \left(0.06858 - 0.04266 \frac{e}{B}\right) \log (B_0 - e_0). \dots \text{II}_\gamma$$

$$(147) \quad + \left[\frac{547.2}{T_0} - 0.4404 - \left(\frac{340.4}{T_0} - 0.2739 \right) \frac{e}{B} \right] \frac{e_0}{B_0 - e_0}. \dots \text{III}_\gamma$$

$$- \left(49.78 \frac{e}{B} + 18.82 \frac{e^2}{B^2} \right) \frac{1}{T_0}. \dots \text{IV}_\gamma$$

The table for II_γ is the same as II_β , page 555, and those for III_γ and IV_γ are given on page 557. Numerical examples can be found on page 575, also on page 697 and following; likewise compare the γ -stage of the Cottage City waterspout, later in this paper, under (c).

(4) The δ -stage relates to a stratum of mixed dry air, frozen aqueous vapor, i. e., ice vapor, and ice or snow below the freezing temperature, and in the present case this stratum extends from the height of about 2880 meters, or 9450 feet, above the sea level to the apex of the cloud which is not far from 4940

meters, or 16,200 feet; so that the δ -stage, or the snow-stage, is about 2060 meters, or 6750 feet in depth, that is about one and one-fourth miles thick. The formula is No. 148 on page 496, in notation of pages 677 or 678, as follows:

$$C_\delta = \text{Constant} = \left(0.2374 + 0.4743 \frac{e}{B} + 0.145 \frac{e^2}{B^2}\right) \log T_{11}. \dots \text{I}_\delta$$

$$(148) \quad - \left(0.06858 - 0.04266 \frac{e}{B}\right) \log (B_{11} - e_{11}). \dots \text{II}_\delta$$

$$+ \left[\frac{547.2}{T_{11}} - 0.4404 - \left(\frac{340.4}{T_{11}} - 0.2739 \right) \frac{e}{B} \right] \frac{e_{11}}{B_{11} - e_{11}}. \dots \text{III}_\delta$$

The numerical tables are on pages 555, 558, and 559, the table for II_δ being identical with that for II_β . Examples of the frozen or δ -stage may be found on pages 576, 697-712, also in the following computations on this waterspout. It is convenient to have a distinct notation for the bottom and top of each of these four stages, and this is done in the notation of pages 677 and 678 of the Cloud Report by simply transferring the suffix of the bottom of the stage to the place of an exponent at the top of that stage. The notation for the top of one stage is equivalent to that of the bottom of the next higher stage, and both symbols may be employed.

In the computation of h , B , t , e for the several stages of the Cottage City waterspout explicit directions will now be given for each stage in the work, for the sake of providing suitable precepts to be followed by others pursuing this line of research, and in order that the several steps may be clearly and accurately understood. The tables for each stage, pages 550 to 559, were all computed so that the absolute temperature, $T = 273^\circ + t^\circ\text{C}$., which appears in the formulæ is found as $t^\circ\text{C}$. in the argument of the tables, for the sake of convenience in computing.

(A) THE α -STAGE OR UNSATURATED PROCESS.

The sea-level conditions as above given may be converted into the metric system by the following table:

	English system.	Transferred to the metric system.	Metric system.
B	30.05 in.	By Smithsonian Table 64 ...	763.27 mm.
t	67.5° F.	By Smithsonian Table 2 ...	19.72° C.
Relative humidity	64 per cent.		64 per cent.
e		By Smithsonian Table 43, e for saturation at 19.72°, Broch's value, is 17.06 mm. and 64 per cent of this is—	10.92 mm.
$\frac{e}{B}$		$10.92 \div 763.27 =$	0.0143
Reduction of $\frac{e}{B}$		From Table 136 p. 690, Cloud Report.	—0.0013
Corrected $\frac{e}{B}$		$0.0143 - 0.0013 =$	0.0130

The result of the discussion of the value of the ratio $\frac{e}{B}$ at the sea level and at the base of the cumulus cloud, as given on pages 677 to 693 of the Cloud Report, was to show that the ratio $\frac{e}{B}$ at the sea level in the α -stage has a larger value than at the base of the cloud. This is due to the fact that in nature the atmospheric process does not exactly follow the adopted theory of the α -stage, and is not a true adiabatic process as the formula requires. We can, however, adapt the formula to this case by applying to the sea-level value of $\frac{e}{B}$ a slight correction to make it smaller at the base of the cloud, and, as the outcome of a very extensive discussion of this problem, the necessary corrections are given in Table 136,

page 690, of the Cloud Report for metric measures, and on Table 18, page 107, of the Barometry Report for English measures. It is necessary to determine this ratio quite accurately for close computations of the humidity correction, according to the theory adopted in these two reports. In this connection the reader is referred to further remarks on this formula, which has been in use since 1897 by the Weather Bureau, as found on page 781 of Dr. J. Hann's *Lehrbuch der Meteorologie*, 1901, and on page 30 of the *Meteorologische Zeitschrift* of January, 1903, by Dr. J. Liznar, where the formulæ are proved to be sufficiently accurate for all practical computations.

The Table 136 gives for the argument $\frac{e}{B} = .0143$ and the assumed approximate height of the base of the cloud, 1100 meters, a correction which I take as $-.0013$, so that we

have $.0143 - .0013 = .0130$ as the value of $\frac{e}{B}$ to be employed throughout the computation, at least so long as precipitation does not produce pseudo-adiabatic conditions. While there is rain falling from the cloud at some distance to the south, where the approximate adiabatic process has been disturbed,

it yet seems proper to retain the full ratio $\frac{e}{B} = 0.0130$ for computing along the line from the waterspout to the apex of the cloud, since it is probable that the activity of the cloud continues still to be nearly adiabatic along the main line of ascension of the vapor of condensation. It is to be particu-

larly noted that this ratio, $\frac{e}{B} = 0.0130$, is to be used in computing every stage, and is the quantity that controls the value of the pressure, temperature, and vapor tension at all the levels of the cloud. The fact that the heights computed by the thermodynamic processes are found to be in close agreement with those measured on the photograph, according to the data of the survey, also tends to justify the adoption of that value of the ratio $\frac{e}{B}$.

The constant C_a in the a -stage is computed as follows:
 I_a 1.3596 Table 95, arguments $\left(t = 19.72, \frac{e}{B} = 0.0130 \right)$.
 $II_a - .4574$ Table 96, arguments $\left(B = 763.3, \frac{e}{B} = 0.0130 \right)$.
 C_a 0.9022 Sum of I_a and $II_a = \text{constant}$.

These values should be determined with accuracy to the fourth decimal place throughout the computation, and the result will then be sufficiently exact to follow the natural processes exactly as they develop. The significance of the constant $C_a = 0.9022$ can be understood from this consideration. We have a set of mutually related thermodynamic quantities, $B = 763.3$, $t = 19.72^\circ$, $e = 10.92$, $\frac{e}{B} = 0.0130$, connected

together in such a way as to give the constant 0.9022.

There are many other sets of values of B , t , e , which can be developed by changing the thermodynamic system adiabatically, that is to say without adding or subtracting any heat. Now, as a given mass of the aqueous vapor, so many grams per thousand grams of air, or so many thousandths of one kilogram, rises from the sea level and ascends upward to the base of the cloud, it evidently passes through pressures and temperatures which are each diminishing in amount. Then at a certain height, where the pressure and temperature are sufficiently reduced, this same mass of vapor will begin to saturate the kilogram, and condense as visible vapor. In all the steps of change in B , t , e from the surface to the cloud-base, the same constant $C_a = 0.9022$ must be retained, and

also the same ratio $\frac{e}{B}$ must be kept, so that there remain the two variables B and t . Now the connection between t and e is fixed at a certain value by the saturation tables, so that if t_1 is assumed, e_1 is found by the table, and B_1 is obtained by computing backwards through Tables 95, 96, the result being checked by

having the ratio $\frac{e_1}{B_1} = 0.0130$. This usually requires a few trials to effect correctly, and one needs a little practise with the data to do it expeditiously. In order to make a beginning with the trial temperatures it is well to remember that in dealing with cumulus clouds the average fall in temperature in passing from the sea level to the base of the cloud, through 1100 meters, is usually 10° to 11° C. This is shown in Table

147, where for the surface temperature $t = 20^\circ$, and $\frac{e}{B} =$

0.0143, which is the original value of $\frac{e}{B}$, we have for the temperature gradient per 1000 meters, $Gt = -9.50$. This table is adapted for the cumulus cloud-base at the height of 1000 to 1100 meters as it stands, without modification. If the temperature is for any other height, the gradient must be modified somewhat as indicated by the subtable on page 727, though the gradients may become very abnormal under certain conditions. It is proper from these considerations to begin the trial values of t_1 , the temperature at the base of the cloud, with $t_1 = 9^\circ$, and $t_1 = 10^\circ$. These values will give resulting values of the $\frac{e_1}{B_1}$ which will enable us by interpolation to find the correct value of t_1 at the third trial.

The following form illustrates the method of procedure for assumed successive values of t_1 . The last column gives the arguments, or the method of obtaining the figures in the second and third columns:

t_1	9°	10°	(Assumed.)
e_1	8.55	9.14	Smithsonian Table 43.
I_a	1.3507	1.3515	$\left\{ \begin{array}{l} \text{Table 95,} \\ \text{arguments.} \end{array} \right\} \left\{ \begin{array}{l} t = 9^\circ, \frac{e}{B} = 0.0130 \\ t = 10^\circ, \frac{e}{B} = 0.0130 \end{array} \right.$
C_a	.9022	.9022	
$II_a -$	.4485	.4493	From preceding result, formula 145.
$C_a - I_a = II_a$			
B_1	670	678	$\left\{ \begin{array}{l} \text{Table 96,} \\ \text{argument.} \end{array} \right\} \frac{e}{B} = 0.0130$.
$\frac{e_1}{B_1}$	0.0128	0.0135	Resulting ratio of $\frac{e_1}{B_1}$.

This ratio should have come out $\frac{e_1}{B_1} = 0.0130$ if the trial values of t_1 had been correct, and interpolation shows that in order to make it so the value of t_1 should be 9.3° .

Repeat the computation,

t_1	9.3°	
e_1	8.72	Smithsonian Table 43.
I_a	1.3509	Cloud Report Table 95.
C_a	.9022	As above.
$II_a -$	.4487	
B_1	672	Cloud Report Table 96.
$\frac{e_1}{B_1}$	0.0130.	$\left\{ \begin{array}{l} \text{This completes the check, and gives } B_1 = 672, \\ t_1 = 9.3^\circ, e_1 = 8.72 \text{ at the base of the cloud.} \end{array} \right.$

We next compute the height at which the visible cloud base floats above the sea level.

For Table 91 we have the formula of barometric reductions and the corresponding height,

$$\log B_0 - \log B_1 = +m - \beta m - \gamma m.$$

B_0 763.27	Logarithm.
B_1 672	2.82737

$$m - \beta m - \gamma m = .05531$$

Since Table 91 is constructed to find m through the arguments, height $= H$ and mean temperature $T_m = \theta$, in order to obtain H by the inverse process we must compute m itself from $m - \beta m - \gamma m = .05531$, by taking $m = .05531 + \beta m + \gamma m$.

By Table 92, with the arguments ($B_0 = 763$ and $e_0 = 10.9$) we have:

$$\text{For argument I} = .378 \frac{e_0}{B_0} = \dots\dots\dots .0054 \text{ and}$$

$$\text{For argument II} = \beta \text{ (for assumed } H = 1100) .0048$$

$$\text{So that } +\beta m \text{ (for } m = .055) \dots\dots\dots + 0.00026$$

$$\text{By Table 93, } +\gamma m \text{ (for } \varphi = 41.5^\circ) \dots\dots\dots + 0.00001$$

$$\text{Hence, since } \log B_0 - \log B_1 = \dots\dots\dots .05531$$

$$\text{We have, } m \dots\dots\dots .05558$$

$$\text{Finally the temperature } \theta = \frac{1}{2}(19.7 + 9.3) = 14.5^\circ.$$

$$\text{By Table 91, with the arguments } (\theta = 14.5 \text{ and } m = 0.05558)$$

$$\text{we find by interpolating } H_a = \begin{cases} 1078 \text{ meters.} \\ 3537 \text{ feet.} \end{cases}$$

We thus find, from the thermodynamic computation, that the height from the sea level to the plane of condensation at the base of the cloud is **3537 feet**. From the measurements on fig. 27, 2d A, as contained on page 311, the approximate height is 60 mm. or 3600 feet. Considering the uncertainty in determining the exact plane which is the base of the cloud sheet, where the vortex motion becomes asymptotic, it is evident that we have reached a satisfactory agreement regarding the height by two independent methods. Furthermore, this check is a good evidence that the fundamental data in these computations are in harmony geometrically and thermodynamically, and we may, therefore, feel confident that the other deductions depending upon them are substantially correct.

The gradients per 100 meters for the α -stage are now easily obtained.

B_0 763.3	t_0 19.7	e_0 10.92	} whence $H_a =$ 1078 meters.
B_1 672.0	t_1 9.3	e_1 8.72	
$B_1 - B_0 = 91.3$	$t_1 - t_0 = -10.4$	$e_1 - e_0 = -2.20$	

Divide $B_1 - B_0$ etc. by 10.78, or the height in units of 100 meters.

$$(G. B)_0 = 8.46 \quad (G. t)_0 = 0.963 \quad (G. e)_0 = 0.204 \dots \text{By observation.}$$

$$(G. B)_c = 8.40 \quad (G. t)_c = 0.950 \quad (G. e)_c = 0.192 \dots \text{Table 147, I, II,}$$

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The tabular gradients are found in Table 147, page 724, which gives the gradients that prevail for average conditions,

by using the arguments ($t_0 = 19.7$ and $\frac{e}{B} = 0.0145$). The agree-

ment is such as to show that the meteorological conditions forming the waterspout cloud are entirely in harmony with those which are found to prevail under similar circumstances, when thunderstorms and tornadoes are in action. If, however, we compare them with the gradients found under the normal August conditions as given in the Barometry Report for Nantucket, we have the following data taken from page 707:

B_m 29.977	t_0 67.7	e_0 .571	} English measures.
B_1 26.470	t_1 60.4	e_1 .445	

Convert these into metric measures.

B_m 761.35	t_0 19.83	e_0 14.50	} Metric measures.
B_1 672.34	t_1 15.78	e_1 11.30	
$B_1 - B_m = 89.01$	$t_1 - t_0 = -4.05$	$e_1 - e_0 = -3.20$	

Divide $B_1 - B_m$ etc. by 10.78, or the height in units of 100 meters.

$$(G. B)_B = 8.24 \quad (G. t)_B = 0.375 \quad (G. e)_B = 0.296$$

The discussion of these gradients will be resumed, as they are important in the theory of atmospheric vortices.

(B) THE β -STAGE, OR SATURATED PROCESS.

The computation of the numerical value of the meteorological elements (B^1, t^1, e^1) at the top of the β -stage, or the bottom of the γ -stage (B_0, t_0, e_0), proceeds as follows:

t_1 9.3° C.	} Taken from the top of the α -stage.
B_1 672 mm.	
e_1 8.72 mm.	
e 0.0130	The same as adopted for the α -stage.
$B_1 - e_1$ 663.28	
$\frac{e_1}{B_1 - e_1}$ 0.0132	

Arguments.

$$\text{I}_\beta = +1.3747 \text{ Table 97 } \left(t = 9.3^\circ, \frac{e}{B} = .0130 \right).$$

$$\text{II}_\beta = -.4419 \text{ Table 98 } \left(B_1 - e_1 = 663.3, \frac{e}{B} = .0130 \right).$$

$$\text{III}_\beta = +.0173 \text{ Table 99 } \left(t = 9.3^\circ, \frac{e_1}{B_1 - e_1} = .0132 \right).$$

$$C_\beta = 0.9501 \text{ Constant for the } \beta\text{-stage.}$$

At the top of the β -stage two quantities are fixed, namely, temperature $t^1 = 0^\circ$ and $e^1 = 4.57$ mm., so that the term I_β can be at once found and subtracted from the constant.

$$t^1 = 0^\circ$$

$$e^1 = 4.57 \text{ mm.}$$

$$\text{I}_\beta = 1.3664 \text{ Table 97, arguments } \left(t = 0^\circ, \frac{e}{B} = .0130 \right).$$

$$C_\beta = 0.9501 \quad C_\beta \text{ taken from above.}$$

$$-0.4163 \text{ Subtract } \text{I}_\beta \text{ from } C_\beta.$$

The term -0.4163 is made up of II_β and III_β at the top of the β -stage, and these depend upon finding by trials B^1 , the pressure there prevailing. It is convenient to employ Hertz' well-known diagram of adiabatic changes in the $\alpha \beta \gamma \delta$ stages, a copy of which is given in Professor Abbe's *Translations, The Mechanics of the Earth's Atmosphere*, page 202, in order to have approximate values to figure the trial computations. Enter the β -stage at the point $B_1 = 672$, $t_1 = 9.3^\circ$, and follow the system of β lines up to $t = 0^\circ$ and $B = 552$. Make two trials by two assumptions of the pressure B^1 .

$$B^1 \quad 552.0 \quad 542.0 \quad \text{Assumed values of } B^1$$

$$e^1 \quad 4.57 \quad 4.57$$

$$B^1 - e^1 \quad 547.4 \quad 537.4$$

$$\frac{e^1}{B^1 - e^1} \quad .0084 \quad .0085$$

$$\text{II}_\beta = -.4289 \quad -.4276 \quad \begin{cases} \text{Table 98, arguments} \\ B^1 - e^1 = 547.4, \frac{e}{B} = .0130 \\ \text{" } 537.4, \text{ "} \end{cases}$$

$$\text{III}_\beta = +.0115 \quad +.0116 \quad \begin{cases} \text{Table 99, arguments} \\ t = 0, \frac{e^1}{B^1 - e^1} = .0084 \\ \text{" " } = .0085 \end{cases}$$

$$\text{II} + \text{III} = -.4174 \quad -.4160$$

The term $\text{II} + \text{III}$ should be equal to $-.4163$, and hence we interpolate another assumed value of $B^1 = 544$. Repeat the trial,

B^1	544	Assumed by interpolation.
$B^1 - e^1$	539.4	
$\frac{e^1}{B^1 - e^1}$	.0085	

Arguments.		
II_β	-.4279	Table 98, $\left(B^1 - e^1 = 539.4, \frac{e}{B} = .0130\right)$
III_β	+.0116	Table 99, $\left(t=0, \frac{e^1}{B^1 - e^1} = .0085\right)$

$II + III = .4163$ The check is complete for $B^1 = 544$.

The computation for H_β , the depth of the β -stage, is as follows:

		Logarithms.			
B_1	672	2.82737	$e_1 = 8.72$	$t_1 = 9.3^\circ$	$\theta = 4.7^\circ$
B^1	544	2.73560	$e^1 = 4.57$	$t^1 = 0$	
$m - \beta m - \gamma m$	0.09177		Arguments.		
βm	+	38	Table 92	$I = .0050,$	$II = .0041$
γm	+	1	Table 93	$e_1 = 8.7$	$H = 1700$
				$B_1 = 672$	$m = .092$
m	0.09216				

$H_\beta = \begin{cases} 1728 \text{ meters.} \\ 5669 \text{ feet.} \end{cases}$ Table 91, arguments ($\theta = 4.7^\circ, m = .09216$).

The gradients for the β -stage are now found,

	mm.	$^\circ\text{C.}$	mm.	
B^1	544	$t^1 = 0$	$e^1 = 4.57$	$H_\beta = 1728 \text{ meters.}$
B_1	672	$t_1 = 9.3$	$e_1 = 8.72$	

$B^1 - B_1 = 128$ $t^1 - t_1 = -9.3$ $e^1 - e_1 = 4.15$

Divide $B^1 - B_1$ etc. by 17.28, or the height in units of 100 meters.

$(G.B_1)_o = 7.40$	$(G.t_1)_o = .538$	$(G.e_1)_o = .240$	By observation.
$(G.B_1)_c = 7.40$	$(G.t_1)_c = .540$	$(G.e_1)_c = .260$	Table 147, IV, for $\frac{e_1}{B_1} = .0130$.

In order to compare this with the normal conditions of the atmosphere in August, we again take the Nantucket data, Barometry Report, page 707, where (B, t, e) are given on the 3500-foot plane and the 10000-foot plane.

The depth of the α -stage = $H_\alpha = 1078 \text{ meters} = 3537 \text{ feet.}$

The depth of the β -stage = $H_\beta = 1728 \text{ meters} = 5669 \text{ feet.}$

Height of top of β -stage above sea level $H^1 = 2806 \text{ meters} = 9206 \text{ feet.}$

By interpolation to the given heights, 1078 meters = 3537 feet and 2806 meters = 9206 feet, we obtain in metric measures, from page 707 of the Barometry Report, the data.

	mm.	$^\circ\text{C.}$		mm.
B^1	548.6	$t^1 = 9.22$	e^1	4.95
B_1	671.6	$t_1 = 15.72$	e_1	11.25
$B^1 - B_1 = 123.0$		$t^1 - t_1 = -6.50$	$e^1 - e_1 = 6.30$	

Divide by 17.28, the height of the β -stage in units of 100 meters.

$(G.B_1)_B = 7.11$ $(G.t_1)_B = .376$ $(G.e_1)_B = .364$

These sets of gradients for the β -stage will also require further discussion.

(C) THE γ -STAGE, OR FREEZING PROCESS.

In the freezing stage there is no change in the temperature, which is $t_0 = 0^\circ$, nor in the vapor tension, which is $e_0 = 4.57 \text{ mm.}$, and we have only to compute the variation in the pressure B_0 .

Constant $II_\beta + III_\beta$	-.4163	brought from the β -stage.
t_0	0°	
B_0	544.	
e_0	4.57	
$\frac{e}{B}$	.0130	

Assume	$\left\{ \begin{array}{l} B^0 \quad 539. \\ e^0 \quad 4.57 \\ B^0 - e^0 \quad 534.4 \\ \frac{e^0}{B^0 - e^0} \quad .0086 \end{array} \right\}$	for the top of the γ -stage.
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Arguments.		
II_γ	-.4273	Table 98 $\left(B^0 - e^0 = 534.4, \frac{e}{B} = .0130\right)$.
III_γ	+.0134	Table 100 $\left(t=0, \frac{e^0}{B^0 - e^0} = .0086\right)$.
IV_γ	-.0024	Table 100 $\left(t=0, \frac{e}{B} = .0130\right)$.

Sum - .4163 This satisfies the check.

For the depth of the γ -stage we compute,

		Logarithms.
B_0	544.	2.73560
B^0	539.	2.73159
$m - \beta m - \gamma m$	.00401	

The corrections for $-\beta m - \gamma m$ can be neglected.

$H_\gamma = \dots \begin{cases} 74 \text{ meters.} \\ 243 \text{ feet.} \end{cases}$ Table 91, argument ($\theta = 0^\circ, m = .00401$)

The pressure gradient in the γ -stage is,

$(G.B_0)_o = 6.76$ By observation.

$(G.B_0)_c = 6.70$ Table 147, V, of the Cloud Report.

(D) THE δ -STAGE, OR FROZEN PROCESS.

The δ -stage extends to the visible tops of the clouds, and we may, therefore, take such a temperature as will, through the intermediate computation, produce a height which agrees with the height measured on the photograph. A preliminary trial of -11°C. for the temperature gives a height that is somewhat lower than the apex of the cloud, and a second trial of -12°C. seemed to be about right, so that it has been adopted for the numerical example.

For the constant in the δ -stage, we have,

t_{11}	0°C.	brought from the top of the γ -stage.
B_{11}	539.	
e_{11}	4.57	
$B_{11} - e_{11}$	534.4	
$\frac{e_{11}}{B_{11} - e_{11}}$	.0086	
$\frac{e}{B}$	.0130	

Arguments.		
I_δ	1.3664	Table 101, $\left(t=0, \frac{e}{B} = .0130\right)$.
II_δ	-.4273	Table 98, $\left(B_{11} - e_{11} = 534.4, \frac{e}{B} = .0130\right)$.
III_δ	+.0134	Table 102, $\left(t=0, \frac{e_{11}}{B_{11} - e_{11}} = .0086\right)$.

δ -Constant .9525

Assume $t^{11} = -12^\circ\text{C.}$

$e^{11} = 1.64 \text{ mm.}$ Table 103, $t = -12^\circ\text{C.}$

$I_\delta = 1.3556$ Table 101, $\left(t = -12, \frac{e}{B} = .0130\right)$.

Constant .9525

$II_\delta + III_\delta = .4031$ This is the constant to determine B^{11} .

Assume B^{11}	415.0	419.0
e^{11}	1.6	1.6
$B^{11} - e^{11}$	413.4	417.4

$\frac{e^{11}}{B^{11}-e^{11}}$	.0040	.0040	
II_{δ}	-.4098	-.4105	$\left\{ \begin{array}{l} \text{Table 98, argument.} \\ \left(B^{11}-e^{11}, \frac{e}{B}=.0130 \right). \end{array} \right.$
III_{δ}	+.0066	.0066	$\left\{ \begin{array}{l} \text{Table 102, arguments.} \\ \left(t=-12^{\circ}, \frac{e^{11}}{B^{11}-e^{11}}=.0040 \right). \end{array} \right.$
	-.4032	-.4039	
Interpolation indicates $B^{11}=414.5$.			
Assume B^{11}	414.5		
e^{11}	1.64		
$B^{11}-e^{11}$	412.9		
$\frac{e^{11}}{B^{11}-e^{11}}$	.0040		Arguments.
II_{δ}	-.4097	Table 98	$\left(B^{11}-e^{11}=412.9, \frac{e}{B}=.0130 \right).$
III_{δ}	+.0066	Table 102	$\left(t=-12^{\circ}, \frac{e^{11}}{B^{11}-e^{11}}=.0040 \right).$
	-.4031	This checks the constant for the pressure $B^{11}=414.5$ mm.	

The depth of the δ -stage is found as follows:

		Logarithms.	
B_{11}	539.0	2.73159	
B^{11}	414.5	2.61752	
			Arguments.
$m-\beta m-\gamma m$	.11407	$\left\{ \begin{array}{l} \text{I} = .0033 (e=4.57, B=539). \\ \text{II} = .0026 (\text{assume } H=2000). \\ \beta m = .0026 \times .114 = .00029. \end{array} \right.$	
$+\beta m+\gamma m$	.00029		
m	.11436		
$\theta = \frac{0-12^{\circ}}{2} = -6^{\circ}$			

$$H_{\delta} = \dots \left\{ \begin{array}{l} 2062 \text{ meters.} \\ 6765 \text{ feet} \end{array} \right. \text{ Table 91 } (\theta = -6^{\circ}, m=.11436).$$

The gradients in the δ -stage are now found:

B^{11}	414.5	t^{11}	-12.0	e^{11}	1.64	$H_{\delta}=2062 \text{ m.}$
B_{11}	539.0	t_{11}	00.0	e_{11}	4.57	
$B^{11}-B_{11}$	-124.5	$t^{11}-t_{11}$	-12.0	$e^{11}-e_{11}$	-2.93	

Divide $B^{11}-B_{11}$ etc. by 20.62, or the height in units of 100 meters.

$$\begin{aligned} (G. B_{11})_o &= 6.04 & (G. t_{11})_o &= .582 & (G. e_{11})_o &= .142 & \text{By observation.} \\ (G. B_{11})_c &= 6.50 & (G. t_{11})_c &= .550 & (G. e_{11})_c &= .140 & \text{Table 147; VI,} \\ & & & & & & \text{VII, Cloud Report.} \end{aligned}$$

The top of the cloud is about 5000 meters above sea level, so that in Section VII, Table 147, for $H=5000$ and $\frac{e_1}{B_1}=.0130$, we find the vapor tension gradient $(G. e_{11})_c = -0.140$ per 100 meters.

Table 51, summary of the data for the Cottage City waterspout, August 19, 1896, contains the results of the preceding computations on the thermodynamic conditions prevailing at that time, in both the metric and the English systems of measures. The first column of figures under each system gives the vertical distances H , measured in millimeters and inches on Hallet's photograph, 2d C, taken at about 1:08 p. m.; the second, the corresponding height in meters and feet; the third, the pressure in millimeters and inches; the fourth, the temperature in degrees centigrade and Fahrenheit; the fifth, the vapor tension in millimeters and inches; the last column the gradients as extracted from the Cloud Report from the same data; also the gradients as deduced from the Baro-

metry Report for the same season of the year. By inspection, it is noted:

(1) In the α -stage in the English measures, the pressure gradient has increased from -0.098 to -0.101 per 100 feet. The normal gradient for August at Nantucket, only a few miles from the scene of the waterspout, is -0.098 per 100 feet, making a pressure fall of -3.47 inches from the sea level to the base of the cloud, or a change of pressure from 30.05 inches to 26.58 inches; the waterspout gradient -0.101 per 100 feet gives a fall of -3.58 inches and a pressure of 26.47 inches at the height 3537 feet, the base of the cloud. There is thus a total change in the vertical gradient of 0.11 inch from the sea level to the cloud, and it is this increased difference of pressure which causes the air to rise generally from the surface of the sea to the cloud, and is intimately concerned with the generation of the vortex tube. We may remark in passing that one of our purposes in constructing tables and charts of the normal values of the pressure, temperature, and vapor tension on the 3500-foot and the 10,000-foot planes, in connection with their values on the sea-level planes, was to afford the data for establishing the *normal vertical gradients* in the lower strata of the atmosphere, which shall represent the average stratification when undisturbed by convectional action. It is very desirable that these gradients should be checked by numerous direct observations of B, t, e , taken day and night, winter and summer, so that the normal gradients can be separated from the convectional gradients in a perfectly reliable manner. As this must be the work of years for the United States, it is permissible to employ the gradients contained in the Barometry Report, which rest upon the best data we now possess, in such discussions as are suggested by the Cottage City waterspout. Similarly, the phenomena of tornadoes in the Mississippi Valley, of thunderstorms generally, of cyclones, anticyclones, and hurricanes can be properly studied only by comparing the abnormal gradients, prevailing in these conditions of rapid convection, with these normal gradients. The differences between these two systems of gradients represent the energy which produces these atmospheric motions, and any explanation of the *cause of the change of the gradients from the normal to the abnormal, or convectional types*, is undoubtedly a direct contribution to the scientific theory of the local circulations of the air. The Barometry Report has furnished us for the first time with the data for constructing the isobars on the higher levels, as is shown on the charts published in the MONTHLY WEATHER REVIEW for January and February, 1903. The series of charts containing the daily isobars on three planes lays bare the mechanism of the dynamic structure in cyclones and anticyclones, so that it is necessary to develop a mathematical analysis in conformity with the observed facts. Furthermore, the same Barometry Report has furnished us with reliable annual residuals, or the variation of the pressure from year to year, and it has been shown in the MONTHLY WEATHER REVIEW for July, 1902,¹ that these residuals synchronize with similar pressure variations over the entire earth, and also to some extent with the annual variations in the frequency numbers of the solar prominences, faculae, and sunspots. These facts suggest the foundation for the true cosmical meteorology of the future. These three lines of study—namely, (1) the abnormal system of convectional gradients, (2) the normal gradients when there are no vertical currents, and (3) the solar-terrestrial synchronous variations—can be developed with accuracy for the United States, and placed upon a strictly scientific basis. Could similar material be computed for several other parts of the earth, the whole cosmical research would be distinctly advanced from an empirical to a thoroughly scientific status. The point of view can be illustrated by employing the data obtained from the Cottage City waterspout, Table 51.

¹ Vol. XXX, pp. 347-354.

TABLE 51.—Summary of the data for the Cottage City waterspout, August 19, 1896.

Stages.	Metric system.					English system.					
	H. photo.	Height.	B.	t.	e.	H. photo.	Height.	B.	t.	e.	Gradient.
	mm.	Meters.	mm.	° C.	mm.	Inches.	Feet.	Inches.	° F.	Inches.	
Upper....	176.4	4942	414.5	— 12.0	1.64	6.95	16,214	16.32	10.4	0.065	
δ-stage.....	Gradients.					Gradients.					
	Range....	73.6	2062	{ — 6.04 — .582 — .142 — 6.50 — .650 — .140		2.90	6,765	{ — .072 — .319 — .00270 — .078 — .302 — .00169			(G) Observed. (G) Cloud.
	Lower....	102.8	2880	539.0	0	4.05	9,449	21.22	32.0	0.180	
γ-stage.....	2.6	74	— 6.76			0.10	243	— .082			(G) Observed.
Upper....	100.2	2806	544.0	0	4.57	3.95	9,206	21.42	32.0	0.180	
β-stage.....	Gradients.					Gradients.					
	Range....	61.7	1728	{ — 7.40 — .638 — .240 — 7.60 — .640 — .260 — 7.11 — .376 — .364		2.43	5,669	{ — .089 — .294 — .00288 — .091 — .294 — .00312 — .085 — .207 — .00437			(G) Observed. (G) Cloud. (G) Barometry.
	Lower....	38.5	1078	672.0	9.3	1.52	3,537	26.46	48.7	0.343	
Upper....			Gradients.					Gradients.			
α-stage.....	Range....	38.5	1078	{ — 8.46 — 0.963 — 0.204 — 8.40 — 0.950 — 0.192 — 8.24 — 0.376 — 0.256		1.52	3,537	{ — 0.101 — 0.631 — 0.00246 — 0.101 — 0.622 — 0.00230 — 0.098 — 0.206 — 0.00355			(G) Observed. (G) Cloud. (G) Barometry.
	Sea level..	0	0	763.27	19.72	10.92	0	30.05	67.5	0.430	

(2) In the temperature of the α-stage the normal gradient is —0.206 F. per 100 feet, and the waterspout gradient is —0.531, which is only a little short of the true adiabatic gradient. The normal temperature fall from the sea level is —7.3°, or a change from 67.5° to 60.2°, at the 3537-foot level. This change from the normal temperature fall (which is small on the Atlantic coast in summer because of the southward bending of the isotherms over the ocean areas) to the adiabatic convectional gradient is a very striking fact. The latter gives a fall of —18.8° F., or a fall from 67.5° to 48.7° F., instead of to the normal temperature 60.2° F., showing that something has occurred to suddenly reduce the normal temperature at the 3537-foot level by the amount 11.5° F.

The cause of this will be explained in the following paragraph.

(3) The normal gradient of the vapor pressure is —0.00355 inch per 100 feet, for Nantucket in August, and this gives a total fall of —0.126 inch, or a change from 0.430 at sea level to 0.304 at the cloud base. The abnormal or convectional gradient prevailing at the waterspout is 0.00246, which makes a change of —0.087, or a diminution of the normal vapor pressure 0.430 to 0.343. Thus, we have a gain of 0.039 in the vapor tension for the waterspout. The result for the α-stage is a total decrease of the pressure by —0.110 inch, a total decrease in the temperature by 11.5° and a total increase in the vapor tension by 0.039 inch. This must be interpreted to mean that the air is rising from the sea level, carrying with it aqueous vapor into levels of temperature about —11.5° F. lower than that which prevails in the normal August weather.

(4) The depth of the α-stage is 3537 feet, of the β-stage 5669 feet, of the γ-stage 243 feet, and of the δ-stage 6765 feet, to the assumed top of the cumulo-nimbus cloud. In the β-stage the pressure gradient changes from the normal rate —0.085, to the convection rate —0.089 per 100 feet. This is equivalent to a fall of —4.82 inches in the normal state, from 26.58 to 21.76 inches; and to a fall of —5.04 in the convection cloud, from 26.46 to 21.42 inches. There is, therefore, a total fall in pressure of —0.227 inch, (— .004 × 56.7 = — 0.227), induced by the change from the normal to the waterspout conditions in the β-stage of the cloud. We have thus a differential gradient per 100 feet in the α-stage of —0.0031, and in the β-stage of —0.0041 inch in favor of a vertical current. The excess in the β-stage over the α-stage of —0.0010 inch may be taken as the effective gradient due to the additional latent heat produced by the condensation of the aqueous vapor to liquid water. This is only one-third the amount of the

gradient due to the cause which produces the general uplift of the air that feeds the cloud. This criterion also proves that there is a more efficient cause for the vertical pressure gradient than the condensation of the aqueous vapor, which has been so generally considered by meteorologists to be the true source of the energy that drives cyclones, following Espy's suggestion of fifty years ago.

(5) The temperature gradient in the β-stage changes, derived for the normal, is —0.207° F. per 100 feet, and —0.294° F. per 100 feet in the cloud. This amounts to a fall of —11.7° F. in 5667 for the normal state, and —16.7° F. for the convectional state, carrying the normal temperature from 60.2° to 48.5° F. in the β-stratum, and from 48.7° to 32.0° F. in the actual β-stage of the waterspout cloud. This is equivalent to a gradient excess in the α-stage of —0.325 of the convection over the normal gradient, and in the β-stage is one-fourth that in the α-stage. In the case of the pressure the excess in the β-stage is four-thirds that in the α-stage. The temperature difference of gradient diminishes rapidly in proportion to the height up to the γ-stage. In that stage and the δ-stage there is not much difference between the normal and the computed convectional gradients. This indicates that the effective cause of a vertical gradient is about exhausted at the height where the isotherms of the 0° is located, or, in other words that the vertical convectional action is properly confined in the waterspout cloud to within about two miles of the ground, and is most active in the lower portion of the cloud. In cyclones this vertical convection is usually limited to within two or three miles of the ground, though the accompanying dynamic action may penetrate into the upper strata as high as three or four miles; in hurricanes the penetration reaches to six or seven miles at least.

(6) In the β-stage the gradient of the vapor tension changes from the normal —0.00437 per 100 feet to —0.00288; the total fall in 5669 feet amounts to —0.248 inch, (—0.00437 × 56.7 = —0.248), in the normal, to —0.163 in the cloud convection, making the fall which should be from 0.304 to 0.056 in the normal state, from 0.343 to 0.180 in the cloud. The difference of gradient is +0.00109 per 100 feet in the α-stage, and +0.00149 in the β-stage, showing that large quantities of vapor are carried upward from the sea level in both stages, but that there is a condensation of aqueous vapor to water equivalent to +0.00040 inch per 100 feet. We can not carry out this comparison between the normal and the convectional gradients in the γ-stage and the δ-stage, but the evidence is that they have become practically identical.

(7) It is desirable to compare these vertical gradients with the commonly observed horizontal gradients. We have in the pressure -0.34 inch in 9206 feet. This is equivalent to -13.47 inches in 364,525 feet, or 111,111 meters, or one degree in the standard latitude of forty-five degrees. Now, on the weather maps, -0.70 inch in five degrees, or -0.14 inch in one degree, is the average horizontal gradient in a highly developed cyclone. Hence, in the convectional cloud formation the vertical gradient is about one hundred times as large as in such horizontal motions. In the temperature the fall of -16.5° in 9206 feet is equivalent to -654° in 111,111 meters, or one degree, and this too is about one hundred times the horizontal gradients which are found on the weather maps. This indicates that *the scale of operations on the horizontal plane is only one hundredth that which occurs in the vertical direction in convectional clouds*. The linear dimensions of cyclones are usually about one hundred on the horizontal to one in the vertical direction, and these two facts, taken together, show how much less force is required to drive a horizontal than a vertical current.

THE CAUSE OF THE FORMATION OF THE WATERSPOUT CLOUD, AND THE VERTICAL CONVECTIONAL VELOCITY.

(1) *Vertical convection due to surface heating*.—We now reach the important question, what was the physical cause of the formation of the cloud, and the vertical convection within it that was the immediate condition of the generation of the vortex tube extending from the base to the ocean? Fortunately, one answer to this question is entirely excluded from our consideration. The disturbance of the normal stratification which produces an abnormal system of gradients and the corresponding vertical currents, may be due to two causes, (1) the surface layers may be overheated relatively to the upper layers, or (2) the upper layers may be undercooled relatively to the surface layers. Either cause would be equally efficient, and it is only a question of which one is actually operating in the case of this waterspout. Overheating the surface layers is due to a perfectly definite physical process, namely, as follows. The effective solar radiation falling upon the earth's atmosphere consists of short wave lengths from 0.30μ to 2.00μ . (Compare figs. 3 and 4, Tables 1 and 2 of my paper on "Solar and terrestrial physical processes," MONTHLY WEATHER REVIEW, December, 1902, Vol. XXX, pp. 562-564.) These short waves, whatever may be the true effective solar temperature at which they are produced, penetrate to the surface of the earth with two sources of depletion, the first, by scattering in the upper atmosphere which cuts out a large percentage of them, and produces the strong glare that is characteristic of the higher layers; the second, by absorption in the aqueous vapor at certain wave lengths, which causes the observed depressions or cold bands in the energy spectrum. There is good reason for believing that the upper as well as the lower atmosphere is heated by the passage of the remainder of the short waves by only a very slight amount, and that this is practically negligible in general discussions. But these short rays falling upon the surface of the earth are readily absorbed, and this absorption powerfully raises the temperature of the land and ocean areas. That practically ends the history of the incoming solar radiation.

The terrestrial radiation, on the other hand, is of an entirely different character, and it has a very different effect upon the earth's atmosphere. The heat radiations at terrestrial temperatures, where the absolute temperature ranges from $T=200^\circ$ to $T=325^\circ$, have wave lengths extending from 2μ to 40μ , and, thus the outgoing wave lengths begin where the incoming lengths end. Many of these long waves, in radiating from the surface of the earth, are quite readily absorbed by the atmosphere, and the heat percolates from the lower through the upper strata by a process of slow conduction and convection.

The aqueous vapor certainly absorbs many waves, as from 4μ to 8μ , and possibly most of the waves beyond 12μ . It was shown very distinctly in my International Cloud Report that the surface temperatures do not diminish with the height at an adiabatic rate, but much more slowly, as is indicated on charts 78 and 79. In Table 162 of the same report is given the number of calories per kilogram required to convert an adiabatic atmosphere into the actual atmosphere as observed. It shows an increase from the ground until the number is about 9.5 calories at the 13,000-meter level in summer and 11.0 calories in winter. This amount of heat may be taken to represent the effect of the outward flowing flux, which, like a slow conduction, keeps the upper atmosphere warmer than it would be if the outward-going waves had the same length as the inward-coming waves. This difference in wave length is the most important factor in the economy of the earth's atmospheric temperatures.

We may now fix our attention more closely upon the changes in the surface temperatures as measured in the normal diurnal and annual periods, and in the local variations of all kinds upon the average conditions. The change in the transparency of the atmosphere due to cloudiness, the difference of altitude of the sun, the character of the surface, whether water area, moist ground, or dry desert, all determine the effective temperature of the surface at any given time. These react upon the corresponding outgoing radiation, which first heats up the lowest strata of the atmosphere, or cools it, according to the prevailing conditions. Strong surface heating by day and cooling by night is, therefore, the regimen to which these layers are subject, and the integral effect of this action in its passage through the upper layers, finally builds up the observed normal gradients of temperature which by no means produce an adiabatic rate of stratification. Temporary convectional currents upward by day, downward by night, upward in some local areas, downward in other areas, constitute the common types of motion due to these causes. The formation of the lower cumulus clouds with moderate convection, of thunderstorms in strong convection, and of desert sand vortices are typical examples of purely surface overheating with vertical convection. But there is an entirely different class of vertical convection, to which sufficient attention has not been paid in meteorological investigations.

(2) *Vertical convection due to the overflow of cold upon warm currents of air*.—It is evident that the vertical convection in the cumulo-nimbus cloud of the Cottage City waterspout could by no possibility have been due to the overheating of the surface by the incoming solar radiation. The phenomenon occurred over the ocean and all the meteorological data of Table 50 show that there was actually no superheating effect near the surface at that time. The prevailing temperature was 67.5° F., while the normal for the month of August at Nantucket was 67.7° F.; and the humidity was 64 per cent, while the normal was 84.3 per cent. *In fact, the 19th of August, 1896, was the driest day of the entire month*, and the powerful vertical convection then taking place could not have been due to the solar radiation acting on the surface conditions. We must, therefore, look for another efficient principle capable of producing the powerful effects shown in the showers preceding the family of waterspouts and the thunderstorm with downfall of hail following them. This we can readily discover by referring to the weather chart of the date, August 19, 1896, fig. 37.

This map shows that a well-defined area of high pressure was just pushing its southeastern front over Vineyard Sound, that the winds were from the northwest, and that there was a fall in temperature of about 15° along the coast line, due to the advance of this cold area. We have, therefore, merely to assume, in accordance with the general fact, that the upper strata are moving eastward in advance of the lower, and that this cold air from the high area was blown forward over Vine-

yard Sound earlier in the strata a mile or two high than at the surface.

A sheet of cold air overran the low, warm, and quiet strata about midday, while the cold air followed at the surface a few hours later, and in these facts we have the exact conditions required to produce the observed powerful convection. Such abnormal cooling of the higher strata is as efficient in producing vertical convectional gradients as a superheating of the surface would be, and the evidence that this was the actual case is so good as to render it a practical proof of this circumstance. The upper strata were cooled suddenly by -12° to -15° F., and this brought showers, the waterspouts, and the thunderstorms in close succession. These were followed later by cooler conditions at the surface, giving a temperature fall from the maximum of the day, 72° , to the minimum, 56.5° , at Vineyard Haven. Under these conditions all the observed facts find so natural and satisfactory an explanation that no further remarks seem to be needed to enforce the theory.

But, it should be noted that this overflow of relatively cold layers of air at a moderate elevation upon the warm surface layers, this forereaching and temporary stratification causes an abnormal system of gradients which produce the vertical currents required to set up the motions that tend to reestablish the normal equilibrium of the atmosphere. This local disturbance of the average gradients, due to the fact that the cold upper air, under certain configurations of the lower currents, is drifted forward upon them, is the *primary cause of most of the phenomena classified as thunderstorms, tornadoes, cyclones, and hurricanes.* In short, all these violent local disturbances of the lower air are largely due to this cause, and this is the true source of the energy expended, though it has been attributed by one school of meteorologists to the latent heat of condensation, and by the other school to the eddies established by differential horizontal velocities. These two latter sources of energy need not be excluded from consideration, for they contribute their quota to the total energy of circulation, but the first cause is the abnormal stratification of the air at moderate elevations. Thus the groups of thunderstorms which frequent the southeastern quadrants of the cyclone are due to the overflowing of the cold northern current upon the warm current from the south. Tornadoes have the same origin and their location shows that they are due to this cause. The cyclone itself is generated by warm currents of air from the Tropics

underrunning the cold sheet which rotates above the surface of the earth, in the hemispherical whirl north of latitude 35° .

The reason for the outflow of warm currents from the Tropics has been indicated in the International Cloud Report, chapters 8 and 10; also there will be found in the MONTHLY WEATHER REVIEW for January and February, 1903, and in the preceding papers of this series, further illustrations and remarks on this theory. The West Indian hurricanes in a similar way are produced in the late summer and autumn by the overflow of the cool upper sheet from the North American Continent upon the warm tropical lower strata, because this sheet is then increasing in size with the southward retreat of the sun. The withdrawal of the sun to the south in fact brings the thermal equator of the higher strata toward the geographical equator earlier than that corresponding to the lower strata. Hence, relatively cold air from the temperate zones at considerable heights, begins to overlay the tropical warm and moist lower strata, and this induces the long continued vertical convection, localized in the hurricane vortex, which in its progressive movement may traverse thousands of miles along its parabolic track. The form of the track is due to the influence of the general circulation localized in centers of action, which builds the south Atlantic high area on the ocean and is manifested in the trade winds, so that the hurricanes usually gyrate along the edge of this special configuration.

The power which is expended for days in succession in a hurricane is due to the fact that the wide expanse of the upper cold sheet covers the temperate zones and overlaps the Tropics at moderate heights. As long as this contrast of temperature, due to abnormal stratification, continues, there is a sufficient source of energy in the resulting thermal engine to produce powerful vertical convection currents, and to sustain the most violent hurricanes, in which the vortex has a depth of several miles in a vertical direction. This theory seems to harmonize completely with what is known about the meteorology of the lower air, and to be such a satisfactory escape from the difficulties of (1) the condensation theory and (2) the dynamic eddy theory, which have always encountered both practical and theoretical objections, that we may expect to find confirmation of it in the future development of the mechanics of the atmosphere.

STUDIES ON THE THERMODYNAMICS OF THE ATMOSPHERE.

By Prof. FRANK H. BIGELOW.

VIII.—THE METEOROLOGICAL CONDITIONS ASSOCIATED WITH THE COTTAGE CITY WATERSPOUT—Continued.

RELATIONS BETWEEN WIND VELOCITIES AND ATMOSPHERIC PRESSURES.

In meteorology there are various relations depending on the influence of wind velocities upon pressure, which must be considered in addition to the usual static barometric pressure used in the construction of synoptic weather maps, and in the determination of heights. Those reductions assume that the air is calm, and that the difference of pressure depending on wind velocity may be neglected. On the other hand, in tornadoes, waterspouts, hurricanes, and strongly developed cyclones the velocity of the wind gives rise to variations of pressure from point to point. In theoretical meteorology, and in the

practical calculation of the effects of high winds upon buildings and other structures, as when a tornado passes over a city or thru a forest, it is very important to have a definite knowledge of the relations between these two phenomena. The literature of this subject is very extensive, but an attempt will be made to bring together in suitable form for reference the facts likely to be of value to meteorologists, engineers, and architects.

FORMULAS FOR WIND VELOCITIES AND PRESSURE GRADIENTS.

Table 52, formulas 1-13, contains the development of the velocity-pressure function from the primary equation (203), page 505, Cloud Report, Vol. II, 1898-1899. By means of the auxiliaries on the side of the table the final equation 13 is found,

$$q = 23.96 \sqrt{\frac{T}{B} \Delta B}.$$

TABLE 52.—Formulas for wind velocities and pressure gradients.

		Auxiliaries.
(1)	$-\frac{\partial P}{\rho} = u \partial u + v \partial v + w \partial w + g \partial z.$	$\frac{1}{\rho} = \frac{1}{\rho_0} \frac{P_0 T}{P T_0}.$
(2)	$-\frac{\partial P}{P} \frac{P_0 T}{\rho_0 T_0} = u \partial u + v \partial v + w \partial w + g \partial z.$	$P_0 = g_0 \rho_0 = g_0 \rho_m B_n.$
(3)	$-\frac{\partial P}{P} = -\frac{\partial p}{p} = \frac{\rho_0 T_0}{P_0 T} (u \partial u + v \partial v + w \partial w + g \partial z).$	$P_0 = g_0 \rho_0 l.$
(4)	$= \frac{1}{g_0 l} \frac{T_0}{T} (u \partial u + v \partial v + w \partial w + g \partial z).$	$\frac{\partial P}{P} = \frac{\partial p}{p} = \frac{\partial B}{B}.$
(5)	$\log p_0 - \log p = \frac{1}{g_0 l} \frac{T_0}{T} \left[\frac{1}{2} (q^2 - q_0^2) + g (z - z_0) \right]$	$\rho_m = 13595.8 \text{ kilograms.}$ $\rho_0 = 1.29305 \text{ kilograms.}$ $l = 7991.04 \text{ meters.}$ $g_0 = 9.806 \text{ meters.}$
(6)	$\log_n p_0 - \log_n p = \frac{1}{156718} \frac{T_0}{T} (q^2 - q_0^2) + \frac{1}{7991.04} \frac{g}{g_0} \frac{T_0}{T} (z - z_0)$	Natural logarithms.
(7)	$\log p_0 - \log p = \frac{1}{360858} \frac{T_0}{T} (q^2 - q_0^2) + \frac{1}{18400} \frac{g}{g_0} \frac{T_0}{T} (z - z_0).$	Common logarithms.

Integrate the velocity term alone when the gravity term is omitted.

(8)	$\int \frac{-\partial B}{B} = \int \frac{1}{g_0 l} \frac{T_0}{T} q \partial q$	Take $-\int \frac{\partial B}{B} = -\frac{1}{B} \int \partial B.$
(9)	$-(B - B_0) = \frac{B}{g_0 l} \frac{T_0}{T} \frac{1}{2} (q^2 - q_0^2).$	
(10)	$\Delta B = B_0 - B = \frac{T_0 B}{2 g_0 l T} (q^2 - q_0^2).$	$\frac{T_0}{2 g_0 l} = 0.001742.$
(11)	$\Delta B = 0.001742 \frac{B}{T} q^2.$	For $q_0 = 0.$
(12)	$q^2 = 574.06 \frac{T}{B} \Delta B.$	
(13)	$q = 23.96 \sqrt{\frac{T}{B} \Delta B}.$	$B \text{ and } \Delta B \text{ in meters.}$ $q = \text{velocity in meters per second.}$

TABLE 53.—Barometric gradient sustaining an eastward velocity only.

		Auxiliaries.
(14)	Standard. $\Delta B = \frac{D}{10514.5} \frac{1}{g_0} 2n v \sin \varphi = 0.1572 v \sin \varphi.$	$D = 111\ 111\ 000 \text{ in millimeters.}$ $g_0 = 9.806 \text{ meters.}$ $2n = 0.000\ 1458.$
(15)	Other density. $\Delta B = 0.1572 \frac{B}{760} \frac{273}{T} v \sin \varphi.$	$B \text{ in millimeters.}$ $v \text{ in meters per second.}$
	$= \frac{1}{17.72} \frac{B}{T} v \sin \varphi = 0.05646 \frac{B}{T} v \sin \varphi.$	

TABLE 54.—The Newtonian theorem.

Auxiliaries.

(1)	$-\frac{\partial P}{\rho} = u \partial u + v \partial v + w \partial w + g \partial z.$	Take $u=v=0.$ $P=g p.$
(16)	$-g \frac{\partial p}{\rho} = w \partial w + g \partial h.$	$z=h.$
(17)	$-\partial p = \frac{\rho}{g} w \partial w + \rho \frac{g}{g} \partial h.$	By integration.
(18)	$p_0 - p = \rho \frac{1}{2g} (w^2 - w_0^2) + \rho (h - h_0).$	When falling from rest $h_0=0. \quad w_0=0. \quad p_0=0.$
(19)	$-p = \rho \frac{1}{2g} w^2 + \rho h.$	
(20)	$w^2 = 2 g h.$	Law of a freely falling body for $p=0.$
(21)	$p_N = \rho \frac{w^2}{2g} A.$	Newtonian theorem. For area $A.$

TABLE 55.—The vertical velocity that just sustains a freely falling body.

Units. Meter-kilogram-second. (M. K. S.)

(22) General formula.	$\Delta B = \frac{1}{574.06} \frac{B}{T} w^2.$	Notation.
(23) Pressure	$\Delta p_A = \Delta B \sigma_m A = \frac{13595.8}{574.06} \frac{B}{T} w^2 A \frac{\text{kilogram}}{m^2}$	w in meters per second.
	$= 23.68 \frac{B}{T} w^2 A.$	A = surface.
	$= 0.08675 B \frac{T_0}{T} w^2 A.$	$\sigma = 1000 \rho$ kilograms.
	$= 0.065936 \frac{B}{B_0} \frac{T_0}{T} w^2 A.$	$\sigma_m = 13595.8.$
		r, D in meters.
		$B, \Delta B$ in meters.

The equivalent normal surface for a sphere $= A = \frac{1}{2} \pi r^2 = \frac{1}{2} \pi \frac{D^2}{4}$. The velocity must be increased by some factor k , as determined by experiments, to allow for the action of the body in modifying the stream lines of pressure. *Ferrel* assumes $k = 1.104$; *Schreiber*, $k = 1.3$.

	($k=1.00$)	Ferrel($k=1.104$)	Schreiber ($k=1.30$)
(24) Pressure	$\Delta p_A = \Delta B \sigma_m \frac{1}{2} \pi r^2 = 37.200 \frac{B}{T} r^2 w^2 k.$	41.07	48.36
	$= 9.300 \frac{B}{T} D^2 w^2 k.$	10.27	12.09
	$= 0.10356 \frac{B}{B_0} \frac{T_0}{T} r^2 w^2 k.$	0.11436	0.13464
	$= 0.02589 \frac{B}{B_0} \frac{T_0}{T} D^2 w^2 k.$	0.02859	0.03366

The weight of the body to be just sustained, σ = specific weight.

(25) Weight	$W_g = \frac{4}{3} \pi r^3 \rho \times 1000 \text{ kilograms} = 4188.8 r^3 \rho = 4188.8 r^3 \frac{\rho_w}{\rho_1}$
	$= \frac{4}{3} \pi \frac{D^3}{8} \rho \times 1000 \text{ kilograms} = 523.6 D^3 \rho = 523.6 D^3 \frac{\rho_w}{\rho_1}$
(26) Specific weight	$\frac{\rho_w}{\rho_1} = \frac{\text{specific weight of body}}{\text{specific weight of water}} = \rho_w$
(27)	$\frac{\rho}{\rho_0} = \frac{\text{density of air above surface}}{\text{density of air at surface}} = \frac{B}{B_0} \frac{T_0}{T}$
(28) For equilibrium	$\Delta p_A = W_g = 37.200 \frac{B}{T} r^2 w^2 k = 4188.8 r^2 \rho_w$
(29)	$= 9.300 \frac{B}{T} D^2 w^2 k = 523.6 D^2 \rho_w$
(30)	$= 0.10356 \frac{B_0}{B} \frac{T_0}{T} r^2 w^2 k = 4188.8 r^2 \rho_w$
(31)	$= 0.02589 \frac{B_0}{B} \frac{T_0}{T} D^2 w^2 k = 523.6 D^2 \rho_w$

TABLE 56.—Velocities.

(32)

$$w^2 = \frac{4188.8}{37.20} \frac{T}{B} \frac{r \rho_w}{k}$$

(33)

$$w^2 = \frac{523.6}{9.300} \frac{T}{B} \frac{D \rho_w}{k}$$

(34)

$$w^2 = \frac{4188.8}{0.10356} \frac{B_0}{B} \frac{T}{T_0} \frac{r \rho_w}{k}$$

(35)

$$w^2 = \frac{523.6}{0.02589} \frac{B_0}{B} \frac{T}{T_0} \frac{D \rho_w}{k}$$

Logs.

$$w = 10.610 \sqrt{\frac{T}{B} \frac{r \rho_w}{k}} [1.02578].$$

$$w = 7.503 \sqrt{\frac{T}{B} \frac{D \rho_w}{k}} [0.87526].$$

$$w = 201.12 \sqrt{\frac{B_0}{B} \frac{T}{T_0} \frac{r \rho_w}{k}} [2.30345].$$

$$w = 142.21 \sqrt{\frac{B_0}{B} \frac{T}{T_0} \frac{D \rho_w}{k}} [2.15294].$$

Units. Centimeter-gram-second. (C. G. S.)

Taking B , r , and D in centimeters, ρ_w in grams, the velocities then become,

(36)

Velocity in meters per second.

$$w = 10.610 \sqrt{\frac{T}{B} \frac{r \rho_w}{k}}$$

(37)

$$w = 7.503 \sqrt{\frac{T}{B} \frac{D \rho_w}{k}}$$

(38)

$$w = 20.112 \sqrt{\frac{B_0}{B} \frac{T}{T_0} \frac{r \rho_w}{k}} = 20.112 \sqrt{r \frac{1}{k} \frac{\rho_0}{\rho} \rho_w}$$

(39)

$$w = 14.221 \sqrt{\frac{B_0}{B} \frac{T}{T_0} \frac{D \rho_w}{k}} = 14.221 \sqrt{D \frac{1}{k} \frac{\rho_0}{\rho} \rho_w}$$

Application of formula (37); w in meters per second.

$w = 7.503 \sqrt{\frac{T}{B}}$							Rain $\rho_w = 1$.		Hail $\rho_w = 0.917$.		Friction factor.	
Height in meters.	B	T					D	$\sqrt{D}$	D	$\sqrt{D \rho_w}$	k	$\sqrt{\frac{1}{k}}$
		273	283	293	303	313						
0.....	cm. 76.00	m. p. s. 14.22	m. p. s. 14.48	m. p. s. 14.73	m. p. s. 14.98	m. p. s. 15.23	cm. 1.00	1.00	cm. 10	3.03	1.0	1.00
1000.....	67.51	15.09	15.36	15.63	15.90	16.16	0.90 0.80 0.70	0.95 0.89 0.84	9 8 7	2.87 2.71 2.53	1.1 1.2 1.3	0.95 0.91 0.88
2000.....	60.25	16.01	16.30	16.58	16.87	17.14	0.60 0.50 0.40	0.78 0.71 0.63	6 5 4	2.35 2.14 1.90	1.4 1.5 1.6	0.85 0.82 0.78
3000.....	53.28	16.99	17.29	17.60	17.89	18.19	0.30 0.20 0.10	0.55 0.45 0.32	3 2 1	1.66 1.35 0.96	1.7 1.8 1.9	0.77 0.75 0.73
4000.....	47.34	18.02	18.35	18.67	18.98	19.29	0.05 0.01 0.001	0.22 0.10 0.03				
5000.....	42.16	19.12	19.47	19.81	20.14	20.47						
6000.....	37.37	20.28	20.65	21.01	21.37	21.72						
7000.....	33.20	21.52	21.91	22.29	22.67	23.06						
8000.....	29.59	22.83	23.24	23.65	24.06	23.44						

TABLE 57.—Conversion factors for units of length, mass, and pressure.

Units.	Meter-kilogram-seconds.		Decimeter-gram-seconds.		Centimeter-gram-seconds.		Foot-pound-seconds.		Inch-grain-seconds.	
	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.
Length, L .	1 meter.	0.00000	10	1.00000	100	2.00000	3.2808	0.51599	39.37	1.59517
	0.1	9.00000	1 decimeter.	0.00000	10	1.00000	0.32808	9.51599	3.937	0.59517
	0.01	8.00000	0.1	9.00000	1 cm.	0.00000	0.032808	8.51599	0.3937	9.59517
	0.3048	9.48402	3.048	0.48402	30.48	1.48402	1 foot.	0.00000	12.0	1.07918
	0.0254	8.40484	0.254	9.40484	2.54	0.40484	0.08333	8.92082	1 inch.	0.00000
Mass, M .	1 kilogram.	0.00000	1000	3.00000	1000	3.00000	2.2046	0.34333	15432.4	4.18843
	0.001	7.00000	1 gram.	0.00000	1	0.00000	0.0022046	7.34333	15.4324	1.18843
	0.001	7.00000	1	0.00000	1 gram.	0.00000	0.0022046	7.34333	15.4324	1.18843
	0.453593	9.65667	453.593	2.65667	453.593	2.65667	1 pound.	0.00000	7000	3.84510
	0.000064799	5.81157	0.064799	8.81157	0.064799	8.81157	0.00014286	6.15490	1 grain.	0.00000
Pressure, $p = \frac{M}{L^2}$.	1 kilo./m. ²	0.00000	10	1.00000	0.1	9.00000	0.20481	9.31135	9.9562	0.99809
	0.1	9.00000	1 gram/dm. ²	0.00000	0.01	8.00000	0.020481	8.31135	0.99562	9.99809
	10	1.00000	100	2.00000	1 gram/cm. ²	0.00000	2.0481	0.31135	99.5620	1.99809
	4.8823	0.68863	48.823	1.68863	0.48823	9.68863	1 lb./ft. ²	0.00000	48.6110	1.68674
	0.10044	9.00189	1.0044	0.00189	0.010044	8.00189	0.020571	8.00000	1 gr./in. ²	0.00000

TABLE 58.—Conversion factors for units of distance, time, and velocity.

Units.	Meters per second.		Kilometers per hour.		Miles per hour.		Feet per second.	
	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.
Distance, S.	1 meter	0.0000	0.001	7.0000	0.00062138	6.79336	3.2809	0.51600
	1000.	3.0000	1 kilometer.	0.0000	0.62138	9.79336	3280.9	3.51600
	1609.315	3.20654	1 609315	0.20654	1 mile.	0.0000	5280.	3.72253
	0.304794	9.48400	0.00030479	6.48400	0.0001894	6.27737	1 foot.	0.00000
Time, T.	1 second	0.0000	0.0002778	6.44370	0.0002778	6.44370	1	0.00000
	3600	3.55630	1 hour.	0.0000	1	0.00000	3600	3.55630
	3600	3.55630	1	0.0000	1 hour.	0.00000	3600	3.55630
	1	0.0000	0.0002778	6.44370	0.0002778	6.44370	1 second.	0.00000
Velocity, $V = \frac{S}{T}$	1 m./sec.	0.00000	3.600	0.55630	2.2369	0.34956	3.2809	0.51600
	0.2778	9.44370	1 kilo./hour.	0.00000	0.6215	9.79336	0.9113	9.95970
	0.4470	9.65034	1.6093	0.20664	1 mile/hour.	0.00000	1.4666	0.16633
	0.3048	9.48400	1.0973	0.04030	0.6818	9.83367	1 ft./sec.	0.00000

TABLE 59.—Conversion factors for pressures and velocities.

Pounds / foot ² .	Miles / hour.	Feet / second.	Meters / second.	Kilometers / hour.
$\Delta p = 1 cv_1^2$		$= (0.6818)^2 cv_2^2$	$= (2.2369)^2 cv_3^2$	$= (0.6215)^2 cv_4^2$
		0.4649	5.004	0.3861
		[9.66734] log	[0.69932] log	[9.58672] log
Kilogr. / met. ² .				
$\Delta p = 4.8823 cv_1^2$		$= 4.882 \times (0.6818)^2 cv_2^2$	$= 4.882 (2.2369)^2 cv_3^2$	$= 4.882 (0.6215)^2 cv_4^2$
		2.270	24.43	1.885
		[0.35597] log	[1.38795] log	[0.27535] log
Grams / cm. ² .				
$\Delta p = 4.8823 \times 0.1 cv_1^2$		$= 0.4882 (0.6818)^2 cv_2^2$	$= 0.4882 (2.2369)^2 cv_3^2$	$= 0.4882 (0.6215)^2 cv_4^2$
		0.2270	2.443	0.1885
		[9.35597] log	[0.38795] log	[9.27535] log
Grains / inch ² .				
$\Delta p = 4.8823 \times 9.9562 cv_1^2$		$= 48.61 (0.6818)^2 cv_2^2$	$= 48.61 (2.2369)^2 cv_3^2$	$= 48.61 (0.6215)^2 cv_4^2$
		48.61	243.3	1.877
		[1.68674] log	[2.38606] log	[0.27346] log
Grams / dm. ² .				
$\Delta p = 4.8823 \times 10 cv_1^2$		$= 48.823 (0.6818)^2 cv_2^2$	$= 48.823 (2.2369)^2 cv_3^2$	$= 48.823 (0.615)^2 cv_4^2$
		48.823	244.32	18.852
		[1.68863] log	[2.38795] log	[1.27535] log

c represents the other terms in the general formula for Δp .

The numbers inclosed in brackets are logarithms of the factors accurately computed.

TABLE 60.—Resistance to a solid moving in a fluid.

Newton's theorem and the coefficient *k*.

ρ = density, h = height, w = vertical velocity, A = Area. The resistance between the solid and fluid is equal to the pressure due to the weight of a column of the fluid $\rho h A = p_N$,

where $w^2 = 2gh$, and $h = \frac{w^2}{2g}$, so that $p_N = \rho \frac{w^2}{2g} \cdot A \dots \dots (21)$.

By observations this requires a coefficient $k = \frac{p_0}{p_N}$.

$$p_0 = \rho \frac{w^2}{2g} k \cdot A$$

On account of viscosity and other forces the more complete formula is $p = aw + bw^2 + cw^3 + \dots$

For air blowing against a plate normally there is an excess of pressure $+ \Delta p_1$ on the front side, and a defect of pressure $- \Delta p_2$ on the back side of the plate.

Take the static pressure of the air on the body $= p$.

$$(40) \text{ Front side pressure } = p_1 = p + \Delta p_1 = p + k_1 \rho \frac{w^2}{2g} A$$

$$(41) \text{ Back side pressure } = p_2 = p - \Delta p_2 = p - k_2 \rho \frac{w^2}{2g} A$$

$$(42) \text{ Wind pressure } = p_1 - p_2 = \Delta p = + \Delta p_1 + \Delta p_2$$

$$= (k_1 + k_2) \rho \frac{w^2}{2g} A = k \cdot \rho \frac{w^2}{2g} A.$$

TABLE 61.—Differential coefficients.

$$\text{From (24) } \Delta p = c \cdot k \frac{B}{T} w^2. \quad c = \text{constant.}$$

k = coefficient of resistance = 1.30.

$$(43) \quad \frac{100 d \Delta p}{\Delta p} = \frac{100 dk}{1.3} = 77 dk. \quad \text{Increase } \Delta k = + 0.1; \\ \text{increase } \Delta p = 7.7 \%$$

$$(44) \quad = \frac{100}{760} dB = 0.13 dB^{mm}. \quad \text{Inc. } \Delta B = + 1 mm; \\ \text{increase } \Delta p = 0.13 \%$$

$$(45) \quad = - \frac{100}{273} dT = - 0.37 dT. \quad \text{Decrease } \Delta T = - 1^\circ; \\ \text{increase } \Delta p = 0.37 \%$$

$$(46) \quad \frac{100 d \Delta p}{\Delta p} = \frac{c \frac{B}{T} w^2 dk \cdot 100}{c \frac{B}{T} w^2 1.3} = \frac{100 dk}{1.3}$$

$$(47) \quad = \frac{ck \frac{1}{T} w^2 dB \times 100}{ck \frac{760}{T} w^2} = \frac{100 dB}{760}.$$

$$(48) \quad = - \frac{ck \frac{B}{T^2} w^2 dT 100}{ck \frac{B}{T} w^2} = - \frac{100 dT}{273}.$$

$$(49) \quad \text{From (24)} \quad w^2 = \Delta p \frac{T}{B} \frac{1}{c.k}.$$

$$(50) \quad \frac{100 \times 2 w dw}{w^3} = \frac{d \left[\Delta p \frac{T}{B} \frac{1}{c.k} 100 \right]}{\Delta p \frac{T}{B} \frac{1}{c.k}}.$$

$$(51) \quad 100 \frac{dw}{w} = \frac{d \Delta p}{2 \Delta p} 100.$$

$$1\% \text{ error in } \Delta p = \frac{1}{2}\% \text{ error in } w.$$

$$(52) \quad = \frac{1}{2} \frac{dT}{273} 100 = 0.18 dT.$$

$$1^\circ \text{ error in } T = 0.18\% \text{ error in } w.$$

$$(53) \quad = - \frac{1}{2} \frac{dB}{760} 100 = -0.07 dB.$$

$$1 \text{ mm error in } B = -0.07\% \text{ error in } w.$$

$$(54) \quad = - \frac{1}{2} \frac{dk}{1.3} 100 = -3.8 dk.$$

$$0.1 \text{ error in } k = -3.8\% \text{ error in } w.$$

TABLE 62.—Coefficient of viscosity for air, μ .

$$(55) \quad \text{Maxwell. } \mu = 0.000\ 000\ 0256 (461^\circ + t) F \frac{\text{pound}}{\text{foot}^2} \cdot \frac{1}{\text{foot}}.$$

The pressure in pounds required to slide 1 square foot of air at the rate of 1 foot per second parallel to a layer 1 foot distant, when the temperature in Fahrenheit degrees is t .

Maxwell. $\mu = 0.000\ 1878 (1 + .00275 t)$, C. G. S. units.

O. E. Meyer. 0.000 1727.

Compare Basset's Hydrodynamics, Vol. II, p. 251.

Coefficient of resistance for air, k .

$$(56) \quad \text{Poncelet and Unwin. } k = a \left[\frac{a}{\beta(a-1)} - 1 \right]^2 \quad \text{Coefficient } k \quad 1.85$$

$$a = \frac{A}{a} = \frac{\text{section of the current}}{\text{section of the body}}$$

$$\beta = \frac{a_1}{a} = \frac{\text{section of contracted current}}{\text{section of the body}}$$

For circular plates.

1.30

Morin. For plates 1 foot square.

1.36

Thibault. For plates 0.3 to 0.5 meter square, velocity 2 to 8 meters per second.

1.83

Duchemin. Reduction of rectilinear motion to circular motion.

p = pressure for rectilinear motion with velocity v .

p_c = pressure for circular motion with velocity v .

A = area of greatest section of the body.

ρ = density of the fluid.

R = arm of rotation at the center of A .

x = distance of center of figure of A from the center of gravity of the half section of A on side of axis.

i = angle of incidence of air striking the front of A .

$\beta = \frac{1}{2} \sqrt{A} \cdot \sin i$ = thickness of the flowing stream on A .

$$(57) \quad p_c = p \left[1 + 1.624 \frac{\beta}{k(R-x)} \right]$$

For plane surfaces. { 1.25
1.30

Mariotte. Plates with low velocities.

1.74

Weissbach. Fixed plates in a moving current of air.

1.25

Woltmann. Experiments at Hamburg, 1785–1790. . . . { 1.19
1.49
1.34

Munche. From experiments of Woltmann, de Borda, Hutton.

1.30

Dubuat. Grashof's discussion.

1.43

Didion. $k = 1.318 + \frac{0.565}{w^2}$.

1.32

Hutton. Sphere for velocities under 90 meters per second, using a ballistic pendulum.

1.40

Nordmark. Double conical bodies.

0.67

Cylindrical bodies.

0.91

Spherical bodies.

0.83

Piobert. } In joint experiments with plates 0.5 to 1.0
Morin. } meter square, velocities 0 to 9 meters
Didion. } per second.

1.357

Poncelet. Recommends as the result of discussing all data available to him.

1.30

De Borda. Whirling machine, plates 0.10 to 0.03 square meter, velocity 3 to 4 meters per second. { 1.39
1.49
1.64

Hutton. Using Robbin's whirling machine, plate 0.01 to 0.02 square meter, and velocities up to 6 meters per second. { 1.24
1.43

Rouse. Whirling machine. Pressure in pounds per foot², $p = 0.00492 v^2$. Velocity in miles per hour.

1.85

Beaufoy. Whirling machine, plate 1 foot square, velocity 2 meters per second, and for a larger plate. { 1.15
1.23

Prechtel. Square plate rotating around one edge as axis

1.90

Rouse. And the Royal Meteorological Society.

1.84

Hagen. Whirling machine with disks which were square, circular, and triangular, the average circumference being 0.50 meter, and velocities between 1.5 and 5.5 feet per second.

For C = circumference of the area $A = 0.50$ meter.

$$(58) \quad \Delta p = (0.00707 + 0.0001125 C) A v^2 \text{ (gram-decimeter second).}$$

$$\Delta p = (0.0707 + 0.01125 C) A v^2 \text{ (kilogram-meter second).}$$

$$\Delta p = (0.0028934 + 0.0001403 C) A v^2 \text{ (} v \text{ in miles per hour, } p \text{ in pounds, } C \text{ in feet, and } A \text{ in square feet).}$$

From these results,

$$k = 1.135 + 0.1805 C = 1.135 + 0.090. \quad 1.225$$

Thiessen and Schellbach. Whirling apparatus with cylindrical bars. D = diameter 2 to 3 millimeters, L = length 0.3 to 1.0 meter.

$$(59) \quad \Delta p = (7.25 v + 0.486 D v^2 + 0.0000698 D^2 v^3) 10^{-6} L, \text{ (C. G. S).}$$

$$\Delta p = (0.0000725 v + 0.0486 D v^2 + 0.0698 D^2 v^3) L, \text{ (M. K. S).}$$

For $B = 760$ millimeters, and $t = 20^\circ \text{C}$,

$$k = 0.00118 \frac{1}{Dv} + 0.7914 + 1.1366 Dv \text{ (for cylinders).}$$

$$k = 0.00154 \frac{1}{Dv} + 1.008 + 1.448 Dv, \text{ (for squares).}$$

$$k = 0.546 \frac{1}{v} + 1.008 + 0.0040 v$$

(for $D = 0.00275$ m).

k for average velocities. 1.300

Thiebault.	Thin plates on a whirling machine.	
	Square plate, $A=0.026$ square meter.	1.525
	Square plate, $A=0.10304$ square meter.	1.784
	Rectangular plate, $A=0.10304$, long side radial.	1.900
	Rectangular plate, $A=0.10304$, short side radial.	1.677
	Square plate, $A=(0.323)^2$, radius=1.370.	1.784
	Square plate, $A=(0.227)^2$, radius=0.966.	1.784
	Square plate, $A=(0.161)^2$, radius=0.685.	1.784
Langley.	Whirling machine, square plate with velocities 4 to 11 meters per second.	1.31
Nipher.	Railroad car direct wind pressures.	1.37
Dines.

FIG. 38.—Special form of whirling apparatus used by Dines.

W = weight of adjustable piece.

ω_0 = angular velocity of rotation of W about C .

The piece $P B W$ is rigid and by its rotation about B assumes a position of equilibrium.

For equilibrium.

$$(60) \quad Px = k \frac{B}{23.68} \frac{r}{(r+x)^2} \omega_0^2 A. x = \pi \cos \phi. y.$$

$$\pi \cos \phi \cdot y = \frac{W(r^2 + y^2) \omega_0^2}{g \sqrt{r^2 + y^2}} \cdot \frac{r}{\sqrt{r^2 + y^2}} \cdot y = \frac{W \omega_0^2 r y}{g}$$

$$(61) \quad k = \frac{T}{23.68 B} \cdot \frac{W}{g} \frac{r y}{(r+x)^2 x} \cdot \frac{1}{A}$$

$$\Delta p \text{ kil. / met.}^2 = \lambda v^2 = \lambda (20.86)^2 \text{ mile / hour} = \lambda (9.3)^2 \text{ meter / sec.}$$

$$k = \frac{\Delta p}{0.06593 v^2} = 0.1754 \Delta p \frac{kq}{m^2} = 0.8564 \Delta p \text{ pound / foot}^2. \quad (65)$$

		λ	k
Squares	4 × 4 inches	0.00347	1.29
	8 × 8	340	1.27
	12 × 12	361	1.34
	16 × 16	350	1.30
Rectangles	16 × 1 inches	0.00391	1.45
	16 × 4	363	1.35
	24 × 6	366	1.36
Circular plates	4.51 inches in diameter	0.00347	1.29
	6.00	338	1.25
	9.03	345	1.28
	13.54	357	1.32

Mean 0.00355 Mean 1.32

$$(62) \quad \Delta p = 0.00355 v^2 \text{ (pound / foot}^2 \text{, and mile / hour).}$$

$$v^2 = 281.7 \Delta p$$

$$v = 16.78 \sqrt{\Delta p}.$$

Stokes.

The resistance to any moving body immersed in a fluid is composed of two parts.

1. That due to viscosity and proportional to v is av .

2. That due to gyratory motions varying with the v^2 and the boundaries of the fluid is bv^2 .

The second may disappear in slow motion but the first remains appreciable.

$\frac{\mu}{\rho}$ = coefficient of viscosity div. by density.

r = radius of sphere.

$$(63) \quad \Delta p = 6 \pi \frac{\mu}{\rho} \cdot \rho r v = 6 \pi \mu r v \text{ inch/second.}$$

Stokes finds $\mu = \rho (0.116)^2$ for air.

The maximum velocity of a falling body becomes permanent when,

$$(64) \quad w_m = \frac{2g}{9} \frac{\rho}{\mu} \left(\frac{\rho_w}{\rho} - 1 \right) r^2 = \frac{2g}{9} \frac{r^2}{\mu} (\rho_w - \rho).$$

ρ_w = density of the sphere.

ρ = density of the fluid thru which it falls.
 g = acceleration of gravity in inches, 386 inches.

w = velocity inch/second:

1 inch/second = 0.0254 meter/second.

For small drops of water at small velocities the viscous resistance of the air is far larger than the impact resistance, as computed by the Newtonian theorem.

For $r = 0.0005$ inch, water $\rho_w = 1$, air in the lower clouds, $\rho = 0.001$, we find $w_m = 1.593$ inch/second = 0.133 foot/second.

Recknagel. Pressure at the center of the front of a plane plate and at the apex of a solid of revolution.

p = pressure in still air surrounding plate, kilogram/meter².

$m = \frac{\sigma_0}{g} = \frac{1.293}{g}$ = mass of 1 cubic meter of air.

v_0 = velocity of the air relative to center of plate, meter/second.

$\frac{C_p}{C_v} = k = 1.41$ = ratio of specific heats.

$$\frac{k-1}{k} = 0.2908.$$

$$p_1 = p \left(1 + \frac{k-1}{k} \cdot \frac{v_0^2}{2g} \cdot \frac{\sigma_0}{p} \right)^{\frac{k}{k-1}} \text{ for any velocities.}$$

$p_1 = p + \frac{1}{2} m v_0^2$ for low velocities.

The pressure diminishes from the center to the edge of the plate.

Schreiber. A discussion of the distribution of the pressure over a flat plate is given on pages 36–38, of Studien über Luftbewegungen, von Paul Schreiber, Abh. d. Kon. Sächs. meteorol. Inst. Heft 3, 1898.

Nipher. A complete experiment of the distribution of pressure over a plate is given in his paper, "A method of measuring the pressure at any point on a structure, due to wind blowing against that structure," by Francis E. Nipher, Transactions of the Academy of Science, St. Louis, Mo., Vol. III, No. 1.

This agrees with the formula, given by Hann on page 11, Über die tägliche Drehung der mittleren Windrichtung, etc. Wien, 1902.

Table 53, formulas 14, 15, deduces the barometric gradient which, acting along the meridian from south to north, will just sustain the wind velocity v directed due eastward. This is the formula for determining the relation between the eastward drift of the atmosphere in the upper strata and the normal gradient which is required to sustain it. This is also found on page 11 of Hann's paper and on page 472 of his *Lehrbuch der Meteorologie*.

Table 54, formulas 16-21, contains the deduction of the Newtonian theorem for the pressure exerted by wind velocity on a body. The general equation becomes

$$(19) \quad -p = \rho \frac{w^2}{2g} + \rho h.$$

In case there is equilibrium between the weight represented by ρh and the pressure exerted by a vertical velocity w^2 , so that $p = 0$, we have

$$(20) \quad \frac{w^2}{2g} = h.$$

which is the law of the velocity for a freely falling body.

Hence, the pressure exerted by the first term, $\rho \frac{w^2}{2g} = p_N$, is the Newtonian pressure.

From observations it is found that this pressure must be multiplied by some factor k , to reduce it to the actual pressure which is exerted upon a rigid body of sensible dimensions. When such a body moves thru a still fluid, or when a moving fluid passes a fixed body, the stream lines of the fluid are deflected in passing the body, making an excess of pressure on the front side, $+\Delta p_1$, and a defect of pressure on the back side, $-\Delta p_2$, so that the total resultant pressure is

$$+\Delta p_1 - (-\Delta p_2) = +\Delta p_1 + \Delta p_2.$$

This is not equal to the Newtonian pressure, but differs from it by some factor, $k = \frac{p \text{ (observation)}}{p \text{ (Newton)}} = \frac{p_0}{p_N}$. The deflection

of the stream lines causes vortices and hydrodynamic pressures of a complicated kind, which are integrated in the total excess of pressure of the positive and the negative types. Many experiments have been made to determine the relations between the front and the back pressure, but they depend largely upon the shape and size of the body and the density of the fluid. The existence of a diminished pressure and consequent inflow, or so-called "suction", on the leeward side of bodies exposed to the wind has been generally recognized,¹ but the experiments of Mr. Irmingier, a Danish engineer, made to determine the amount of such suction, shows it to be present to an unexpected extent. His measurements were made by the use of hollow plates and models of thin sheet iron exposed in an air duct $4\frac{1}{2}$ by 9 inches cross section, at various angles and positions, to velocities ranging from 16 to 32 miles per hour. We quote the data from Julius Baier's paper on wind pressures in the St. Louis tornado, American Society of Civil Engineers, Vol. XXXVII, 1897, No. 805. (See Table 63.)

This shows that the percentages vary widely with the shape of the body, and its exposure to the direction of the wind, and that the lee suction is often much in excess of the front pressure. For spheres the front pressure is 28 per cent and the back suction 72 per cent of the total pressure p . The total pressure on a sphere is only 57 per cent of that on a thin plane having each side equal to the diameter of the sphere. The subject is very complex in application to special cases.

¹See Abbe in the Monthly Weather Review, November, 1886, Vol. XIV, p. 332, and his publication of observations on the pressure and suction around the Weather Bureau station at Mount Washington, N. H., in his *Meteorological Apparatus and Methods*, pp. 142-144. See also the results of experiments on chimneys and cowls, Proc. Am. Acad. Arts and Sciences, Boston, 1848; Journal Franklin Institute, Philadelphia, 1842.

TABLE 63.—Percentage of front and back pressures (Irmingier's results).

	PRESSURE IN A HORIZONTAL DIRECTION	PRESSURE ON THE WINDWARD SIDE %TOTAL PRESSURE	SUCTION ON THE LEEWARD SIDE %TOTAL PRESSURE
→ [∞]	p	45	55
$0.95p$	57	43
$0.79p$	24	76
$0.57p$	28	72
$0.25p$	18	82
$0.59p$	58	42
$0.42p$	14	86
$0.71p$	63	37

The wind velocities are usually taken in the United States by the Robinson anemometer, but the indicated velocities must be reduced about 20 per cent, in order to obtain true values of the velocity to enter into the formula.

The reduction factor from miles per hour to meters per second is as follows:

$$1 \text{ mile per hour} = 0.4470 \text{ meter per second.}$$

MARVIN'S CORRECTION TO OBSERVED WIND VELOCITIES.

The velocity of the wind is very generally measured by some form of the Robinson cup anemometer. From early experiments it was found that the distance passed over by the center of one of the revolving cups would, if multiplied by three, give the velocity of the wind, and the wheels and recording dials of the instrument were geared to read wind velocities directly by taking into account this factor of reduction. Later experiments have shown that with anemometers of the size commonly used this ratio is erroneous, and the indicated velocities are about 20 per cent too great, but no change has been made in the recording device and the "wind observations published by the various meteorological institutions at the present time have only a relative, but not absolute, value. It is very probable that many experiments on the relations of wind velocities to wind pressures have been made in which this anemometer correction has not been properly applied".

Professor Marvin has determined the correction to be applied to the readings of the standard form of anemometer used by the Weather Bureau. It is expressed by a logarithmic formula from which the following table taken from his report on wind pressures is computed:

TABLE 64.—Corrected wind velocities as indicated by a Robinson anemometer, in miles per hour.

Indicated velocity.	0	1	2	3	4	5	6	7	8	9
0	9.6	10.4	11.3	12.1	12.9	13.8	14.6	15.4	16.2	17.0
10	17.8	18.6	19.4	20.2	21.0	21.8	22.6	23.4	24.2	24.9
20	25.7	26.5	27.3	28.0	28.8	29.6	30.3	31.1	31.8	32.6
30	33.3	34.1	34.8	35.6	36.3	37.1	37.8	38.5	39.3	40.0
40	40.8	41.5	42.2	43.0	43.7	44.4	45.1	45.9	46.6	47.3
50	48.0	48.7	49.4	50.2	50.9	51.6	52.3	53.0	53.8	54.5
60	55.2	55.9	56.6	57.3	58.0	58.7	59.4	60.1	60.8	61.5
70	62.2	62.9	63.6	64.3	65.0	65.8	66.4	67.1	67.8	68.5
80	69.2									
90										

Total velocity = number on side and number on top of same column. Thus, for 41 the corrected velocity is 34.1; for 57 the corrected velocity is 45.9.

Table 55, formulas 22-31, contains the formulas for the vertical velocity that just sustains a freely falling body. The successive steps are clearly indicated. In formula 24 the coefficients are computed for the value of $k = 1.3$ employed by Professor Schreiber. Since the weight of a body, $W_g = \frac{4}{3}\pi r^3 \rho \times 1000$ kilograms, must be put equal to $\Delta p_A = \Delta B \sigma_m \frac{1}{2} \pi r^2$ for equilibrium under the specified conditions, further formula can be developed. The area A in (23) is

taken as equivalent to half the greatest cross section, so that $A = \frac{1}{2} \pi r^2 = \frac{1}{2} \pi D^2$. According to Irmingers tests the coefficient of πr^2 should be 0.57 instead of 0.50. The specific weight of the body $= \rho_w = \frac{\rho_w}{\rho_1}$, where the specific weight of water is the unit. The density of the air at any point is found by the usual formula $\rho = \rho_0 \frac{B}{B_0} \frac{T_0}{T}$. Formulas 28-31 contain the several relations found by putting (24) in combination with (25).

Velocities.

Formulas 32-35 of Table 56 contain the resulting velocities in the kilogram-meter-second system (M. K. S.). In the centimeter-gram-second (C. G. S.) system for the barometric pressure B , the radius r , and the diameter D in centimeters, for the density ρ in grams, and the velocity in meters per second, we have the values of the velocity w , as given in formulas 36-39. Formula 36 gives the velocity in meters per second in terms of the absolute temperature T , the barometric pressure B in centimeters, the radius of sphere in centimeters, the specific weight of the body in grams. The coefficient k must be assigned from experimental data. Formula 37 employs the same data as 36 except that the diameter is used instead of the radius. Hence, $10.61 = 7.503 \times \sqrt{2}$ and $20.112 = 14.221 \times \sqrt{2}$.

In 38 and 39 the substitution $\frac{B_0}{B} \frac{T}{T_0} = \frac{\rho_0}{\rho}$ is made, in case one prefers to use the densities rather than the pressures and temperatures. We have taken formula 37 for development and application to the formation of hail.

Formula 37, $w = 7.503 \sqrt{\frac{T}{B} D \rho_w \frac{1}{k}}$. The first section of the table contains $w = 7.503 \sqrt{\frac{T}{B}}$ for the arguments T and B . T begins at 273° , since in hail formation the temperature does not fall below 0° C., and it extends to $T = 313^\circ$, which is the extreme of summer heat at the surface. Approximate values of B are taken at the heights H as indicated, so that H or B may be used in practical applications. If $D = 1$ centimeter, $\rho_w = 1$ for water, and $k = 1$, the velocity in meters per second is found in the body of the table. In application to water drops of various sizes, large drops 0.70 to 0.50 cm., common drops 0.40 to 0.20 cm., fine drops 0.100 to 0.001 cm., the subjoined multiplying factor $\sqrt{D}$ must be used. In the case of hail, for $\rho_w = 0.917$, the tabular factor ranges from $D = 10$ cm. to $D = 1$ cm. giving the several values of $\sqrt{D \rho_w}$.

It is shown in the following tables that the coefficient k is equal to about 1.30 for bodies of considerable dimensions, such as engineers naturally employ, plates, disks, spheres, cubes, and parallelepipeds placed on whirling machines for tests. These can hardly be true for water drops which are not rigid, but very flexible in their form while falling, since they undergo periodic changes in shape when in motion, like oil drops rising in water. It is difficult to assign values to k for fluids, and on that account the factor has been kept separate from the first section of the table. My opinion is that $k = 1.1$ nearly for water, but that on the solidification into hail, the value of k approaches $k = 1.30$, in consequence of its rigid shape in the solid state, which is of dimensions comparable with those used in the physical experiments. These tables can be readily used in numerous combinations, by selecting the suitable sets of factors to be multiplied together. The effect of k is to diminish the required velocity w in meters per second, since the form of the body generates a term which is itself equivalent to an addition to the actual

wind velocity. The required velocity increases with the dimensions of the body, and with the increase of density of bodies of the same dimensions.

Conversion factors.

Tables 57, 58, and 59 contain the conversion factors between several systems of units which are found in the papers relating to the velocities and pressures. Table 57 gives the units of length L , the units of mass M , the pressure $p = \frac{M}{L^2}$. Table 58 gives the distance S traversed in the unit of time, the unit of time T , the velocity $V = \frac{S}{T}$. Table 59 gives various combinations between the pressure and the velocities for several systems of units. For example, the pressure in pounds per square foot with the velocity in miles per hour, becomes the pressure in grams per square centimeter with the velocity in meters per second, by multiplying with the factor 2.443 (logarithm = 0.38795). The inverse reduction is performed by multiplying with $\frac{1}{2.443} = 0.4093$ (logarithm = 9.61205).

Resistance to a solid moving in a fluid.

The problems in hydrodynamics relating to solids moving in a fluid, or to a fixed solid in a moving fluid, are very numerous and their discussion can be found in several treatises. In Table 60, formulas 40 to 42, the Newtonian theorem is resumed in connection with the factor k . But it was shown that there is a factor k_1 for the front side of a plate or solid of any form, and a factor k_2 for the back side of this body. The wind pressure is made up of two parts $\Delta p = \Delta p_1 + \Delta p_2$, and the factor k of two parts $k = k_1 + k_2$. These must be determined for special objects, and their values can not be assigned from general considerations.

Differential coefficients.

The variations of pressure Δp depend upon changes in the barometer height ΔB , the temperature ΔT , the coefficient Δk , and the velocity Δw . Table 61, formulas 43 to 54, contains the differential coefficients, and the corresponding changes of the terms in percentages. Thus—

$$\begin{aligned} + 0.1 \Delta k &= 7.7 \% \Delta p. \\ + 1^{\text{mm}} \Delta B &= 0.13 \% \Delta p. \\ - 1^\circ \Delta T &= 0.37 \% \Delta p. \\ 1 \% \text{ error in } \Delta p &= 0.5 \% \text{ error in } w. \\ 1^\circ \text{ error in } T &= 0.18 \% \text{ error in } w. \\ 1^{\text{mm}} \text{ error in } B &= 0.07 \% \text{ error in } w. \\ 0.1 \text{ error in } k &= 3.8 \% \text{ error in } w. \end{aligned}$$

These values of the ratios are convenient in estimating the mutual changes which take place among these quantities.

The resistance coefficient k .

It is evident that an error in the value of the resistance coefficient k is much more efficient in producing an error in the wind velocity as computed than are similar errors in the determination of the barometric pressure and temperature. It is, therefore, important to discuss the coefficient k with much care, and to pass in review the results of the experiments which have been executed for the purpose of establishing its value under different conditions. These involve bodies of different sizes and shapes, carried thru fluids, such as air and water, with variable velocities. The experiments extend thru the past century, and many of them have been executed with all possible care as to details, in order to secure scientific precision. Summaries of these studies may be found in Abbe's²

²Treatise on Meteorological Apparatus and Methods, by Cleveland Abbe, Appendix 46, Annual Report Chief Signal Officer, 1887, pages 218-240. Mechanics of the Earth's Atmosphere, C. Abbe, Translation of Hagen's paper, Smith. Misc. Coll., 843, 1891.

and in Schreiber's³ compilations to which I am chiefly indebted for the accompanying data. The original papers are enumerated therein, and the bibliography need not be repeated in this place.

Table 62, formulas 55 to 65, contains the summary of values of k , together with a brief statement regarding the nature of the experiments and the fundamental formulas of the instrumental work. Formula 55 gives Maxwell's value of the coefficient of viscosity in British units and in C. G. S. units, with reference to Basset's Hydrodynamics. Formula 56 gives Poncelet and Unwin's equation, which takes account of the contraction and expansion of the stream lines in passing by a body. Formula 57 gives the equation for transforming the pressure on a body moving rectilinearly into that encountered by it when carried on a whirling machine. The group of equations under formula 58 contains Hagen's results in (C. G. S.), (K. M. S.), and (P. F. S.) units, respectively. It is to be observed that a considerable factor depends upon the circumference of the plates, and that the value of $k = 1.104$, when the circumference is very small, as in raindrops. Formula 59 contains the result of Thiessen's and Schellbach's experiments on long cylindrical rods whirled by a machine, and it shows that there is a complex function depending upon the first and second powers of the velocity which is involved in the coefficient of resistance. Formula 60 gives the equation for equilibrium in Dines's machine, which has a special device for determining the pressure (fig. 38). The rectangular arm PBW is rigid and rocks upon the axis B ; the arm BW is stayed between two stops, with electric contact, so that the length of the working arm for W can be accurately adjusted to the whirling pressure P on the plate A . Formula 61 gives the equation for k and the accompanying table of values for λ and k . λ is the coefficient required to find the pressure in pounds per square foot from the velocity in miles per hour, and it averages $\lambda = 0.00355$, which appears in formula 62, $\Delta p = 0.00355 v^2$. Professor Marvin has established the value for the Weather Bureau $\lambda = 0.00400$, from which $\Delta p = 0.00400 v^2$. Professor Nipher has determined the value $\lambda = 0.00251$ on the windward side alone, so that from Irmingier's experiments we are safe in taking the total value as that given by Dines or Marvin for general conditions. The result of Stokes's investigation on the resistance of any moving body immersed in a fluid shows that it consists of two parts, the first due to viscosity proportional to the velocity, and the second due to the gyratory motions which are generated in the fluid under the existing conditions, proportional to the square of the velocity. Formula 63 gives Stokes's equation and the value of μ for air, with a velocity v in inches per second. There are, however,

other methods of determining the viscosity which are considered better, referred to in 55. Formula 64 gives the maximum velocity of a falling body when it has become uniform, with an example for a raindrop in the air. Since the pressure on a large plate varies between the center and the edge by certain laws to be discovered by experiment, the distribution of the pressure has been discussed by Recknagel, Schreiber, and Nipher. Formula 65 contains Recknagel's equation for any velocities, the terms being specified. For Schreiber's and Nipher's results reference may be made to their papers.

It will be seen by inspecting the catalog of values for k , the coefficient of resistance to a rigid body moving in air, since the water experiments have not been included in the list, that there is a great diversity of data to be considered in selecting a mean value. This arises from the great variety of conditions involved in the investigations, and also from the great length of time covered in the researches, about 100 years. At the same time it is evident that they average closely to the value assigned by Schreiber, $k = 1.30$. This value applies generally to rather large objects, plates, disks, and solids having a normal sectional area of one square foot or more. For small bodies, such as drops of rain or even hailstones, I am inclined to believe that Ferrel's value of k is nearly correct, that is $k = 1.10$. The uncertainty, however, is such that it must be left to the investigator to make his choice as to the adopted value. Consequently in Table 56 I have kept the value of k apart from the section containing the vertical velocity w . If we choose for hailstones an average diameter of one centimeter, I suppose that nine-tenths of the value of w given in the column under 273 at the several heights is about what may fairly be expected as the sustaining velocity in meters per second up to a height of 8000 meters. That is, the value of w at the freezing temperature and the height at which hail forms is as in the following table:

TABLE 65.—Probable sustaining vertical velocities, w , for hailstones.

Height.	Probable sustaining velocity.	
	For common hailstones.	For very large hailstones.
Meters.	M. per s.	M. per s.
0	12.8	25.6
1000	13.6	27.2
2000	14.4	28.8
3000	15.3	30.6
4000	16.2	32.4
5000	17.2	34.4
6000	18.3	36.6
7000	19.3	38.6
8000	20.5	41.0

³ Studien über Luftbewegungen, von Paul Schreiber, Adh. d. Kön. Sächs. Met. Inst. Heft 3, Chemnitz, 1898.

STUDIES ON THE THERMODYNAMICS OF THE ATMOSPHERE.

IX.—THE METEOROLOGICAL CONDITIONS ASSOCIATED WITH THE COTTAGE CITY WATERSPOUT—Continued.
THE MAXIMUM FALLING VELOCITY FOR RAIN IN THE LOWER ATMOSPHERE.

The velocity in meters per second which just sustains a freely falling body in the atmosphere has been computed by formula 37, Table 56 :

$$w=7.503\sqrt{\frac{T}{B}D\rho_w\frac{1}{k}},$$
where T is the absolute temperature, B the barometric pressure in centimeters, D the diameter in centimeters, ρ_w the density of the falling body in grams per cubic centimeter, and k the so-called friction factor, ranging from 1.0 to 1.9. In view of the many combinations which can occur between these terms, an extensive series of tables would be required to express the results in final values of w , the velocity in meters per second. I have, therefore, in Table 56, separated the formula into its three constituent parts, for convenience,

$$w=7.503\sqrt{\frac{T}{B}}, \quad \sqrt{D\rho_w} \quad \text{and} \quad \sqrt{\frac{1}{k}}.$$

The reader should select the values that are appropriate for the case in hand and make the necessary multiplications in order to find the maximum falling velocity. This may be properly illustrated by the example of rain, which will require some further modifications, because of the facility with which water in the atmosphere can take on different sizes, since drops occur ranging in diameter from 0.01 to 7 mm. In the case of rain $\rho_w=1.0$, and we select $k=1.0$, tho it will be shown that this is correct only for common drops from 0.20 to 1.50 mm. in diameter. For convenience we take

$w=7.503\sqrt{\frac{T}{B}}=15.0$ m. p. s., since this is the value prevailing in the lower strata of the atmosphere at common temperatures. In the Meteorologische Zeitschrift for June, 1904, Prof. P. Lenard of Kiel has a valuable article on rain, which will be used in this connection as it affords some experimental data of importance and is generally in agreement with the results of the discussion already described in the preceding pages.

Table 66, Bigelow's formula for rain, contains the computation for rainfall, where the drops are divided into three classes:

- I. Diameter, $D=2r$, from 0.01 to 0.20 mm.
- II. Diameter, $D=2r$, from 0.30 to 0.50 mm.
- III. Diameter, $D=2r$, from 1.00 to 5.50 mm.

Since the C. G. S. system of units is employed the values of $D=2r$ must be exprest in centimeters, as in the second column; the third column contains $\sqrt{D}$, and the fourth $w=15\sqrt{D}$, which is the required velocity in meters per second.

TABLE 66.—Bigelow's formula for rain.

$w=7.503\sqrt{\frac{T}{B}D\rho_w\frac{1}{k}}.$			
For rain in lower atmosphere, take $\rho_w=1.0$ for water, and $k=1$.			
Take $7.503\sqrt{\frac{T}{B}}=15.0$ m. p. s. approximately.			
I. FOR FINE DROPS, 0.01 to 0.20 mm.			
$D=2r$	$\sqrt{D}$	$w=15\sqrt{D}$	$w=\frac{2g}{9\mu}r^2$
mm. cm.		m. p. s.	m. p. s.
0.01 = 0.001	0.032	0.48	0.0032
0.02 = 0.002	0.045	0.67	0.0127
0.03 = 0.003	0.055	0.83	0.029
0.05 = 0.005	0.071	1.07	0.079
0.10 = 0.010	0.100	1.50	0.32
0.20 = 0.020	0.142	2.13	1.27

II. FOR COMMON DROPS, 0.30 to 0.50 mm.

$D=2r$	$\sqrt{D}$	$w=15\sqrt{D}$	$w^2=\frac{g}{\gamma\rho}\cdot r$
mm. cm.		m. p. s.	m. p. s.
0.30 = 0.030	0.173	2.60	2.73
0.40 = 0.040	0.200	3.00	3.15
0.50 = 0.050	0.223	3.35	3.53

III. FOR LARGE DROPS, 1.00 to 5.50 mm.

$D=2r$	$\sqrt{D}$	$w=15\sqrt{D}$	Ob n.	$\sqrt{\frac{1}{k}}$	k
mm. cm.					
1.00 = 0.100	0.316	4.74	4.40	0.98	1.1
1.50 = 0.150	0.387	5.81	5.70	0.94	1.1
2.00 = 0.200	0.447	6.71	5.90	0.88	1.3
2.50 = 0.250	0.500	7.50	6.40	0.85	1.4
3.00 = 0.300	0.548	8.22	6.90	0.84	1.4
3.50 = 0.350	0.592	8.88	7.40	0.83	1.5
4.00 = 0.400	0.632	9.48	7.70	0.81	1.5
4.50 = 0.450	0.671	10.07	8.00	0.79	1.6
5.00 = 0.500	0.707	10.61	8.00	0.76	1.8
5.50 = 0.550	0.742	11.13	8.00	0.72	1.9

For fine drops, I, w ranges from 0.48 to 2.13.
For common drops, II, w ranges from 2.60 to 3.35.
For large drops, III, w ranges from 4.74 to 11.13.
These results are obtained by applying the same law for all drops, from the finest to the largest which occur in the atmosphere. This must, however, be modified for fine drops, I, and for large drops, III, in order to conform to the experimental observations. When the drops are very fine the viscous resistance of the air becomes the prevailing force that holds them from falling, and by formula 64, Table 62, the maximum falling velocity is permanent for

$$w_m=\frac{2g}{9}\frac{\rho}{\mu}\left(\frac{\rho_w}{\rho}-1\right)r^2=\frac{2g}{9\mu}(\rho_w-\rho).$$

By taking $(\rho_w-\rho)=1.00-0.00129=1.00$, this becomes $w_m=\frac{2g}{9\mu}r^2$, which is the formula employed by Lenard for fine drops. In this $g=981$ cm., $\mu=0.00012$, by formula 55, Table 62, and r is taken in centimeters. The result in the fifth column ranges for fine drops from 0.0032 to 1.27 m. p. s., and they are much smaller than those given by the general formula.

In the case of common drops, Lenard uses the formula $w^2=\frac{g}{\gamma\rho}r$, where $g=981$ cm., $\rho=0.00129$, and $\gamma=0.153$, a constant derived by experiment. The results range from 2.73 to 3.53, and are in close agreement with the general formula, where $k=1.0$. This shows that the formula for impact resistance as distinguished from viscous resistance begins to be applicable for drops whose diameters are about 0.25 mm. The fact that $k=1.0$ indicates that the common drops do not experience any deformation relatively to the passing stream lines, the surface tension being strong enough to simply adjust the shape of the drop to the curvature required for avoiding any resistance due to the shape of the body, except that tangential to the surface of the drops. When the drops increase in size beyond 0.50 mm. a deformation sets in which it is important to describe more fully.

Professor Lenard's experiments on large drops, with diameters 1.00 to 5.50 mm., were conducted by means of a machine which produced a vertical current whose velocity could be regulated and measured. In the midst of this the water drops falling from above were made to float, and their sizes when in equilibrium were studied. The resulting velocities for corresponding diameters are given in column 5 of section III, and they range from 4.40 to 8.00 m. p. s. It was seldom that larger drops than 5.50 mm. survived without breaking up, and the

maximum current was 8 m. p. s. If we divide this experimental value of the vertical velocity by the computed velocity

in column 4, we find the ratios $\sqrt{\frac{1}{k}}$ in column 6, section III.

We take out the corresponding values of k which are placed in the last column of Table 66, where they range from 1.1 to 1.9. We may, therefore, render the general formula for w applicable to raindrops of different sizes by taking $k=1.0$ for common drops, and gradually increasing its value for large drops from $k=1.1$ to $k=1.9$, as indicated in the table.

In his experiments on the deformation of raindrops in a vertical current of air, which could be well observed, Lenard found that the first effect of the current on the shape of the drop was to flatten it so that the axis parallel to the direction of the current was shortened. A further increase of velocity produced an increase of surface friction along the meridians of the drop, which also set up oscillations, and gradually produced vortex ring motion around a circle in the outer portion of the drop, lying in a plane perpendicular to the motion of the current. This vortex ring then separated into a corona of beads, the ring breaking up into smaller drops which became individuals, and broke up the large drop into fine drops. The fine drops began to increase in size by means of two processes, (1) collisions of drops in the current, (2) the attraction of drops by means of the electric charges which always accompany the aqueous vapor in its various stages of ionization. Lenard made counts for the number of drops of different sizes occurring in several rains and found that the number of fine drops is greatly in excess of that for large drops. The series of assorted sizes does not change regularly from the smallest to the largest, but they accumulate in groups, some sizes being entirely wanting, tho the number of large drops in any group is not so great as in the groups of small diameters. There is a continual interchange in the sizes and numbers in each group because of the growth, deformation, and separation of large into small drops. It is evident that the true physical values of the surface tension in drops can be obtained by computations on such data as that found in this manner. In the quiet air of the Tropics where the aqueous vapor content is great the drops may sometimes grow to a diameter of 7.0 or 8.0 mm., tho that is not common. The time of oscillation in the process of deformation and disintegration is apparently two or three seconds. The subject of rain invites to more exact experimental research than has yet been bestowed upon it.

THE PROBABLE VERTICAL VELOCITY IN THE CLOUD.

It is desirable to obtain some idea of the vertical velocity within the cloud itself for the purpose of judging of the validity of certain theories which have been proposed to account for the formation of heavy hailstones. The formula to be employed is

$$w^2 = 574.06 \frac{T}{B} \Delta B,$$

in the adopted system (M. K. S.) where B and ΔB are in meters, tho, since they occur in the ratio $\frac{\Delta B}{B}$, they can be taken in

millimeters; w is the velocity in meters per second. Referring to the data for the waterspout in Table 51, we find the quantities available for the discussion. It is evident that there is no difficulty regarding T or B , but that the value to be assigned to $\Delta B = B_0 - B$ is very uncertain. At first I take B_0 as the static pressure in the normal system of gradients as obtained from the Barometry Report, and B the pressure computed as above from the observed cloud conditions.

α -stage.

(See Table 51. Summary of data.)

The pressure at sea level is 763.27 mm.
The gradient in the static state is -8.24 mm/100 m.
The height is 10.78×100 meters.
The pressure fall is -88.83 mm.

B_0 = pressure at cloud base in static state 674.44 mm.
The gradient in the convection state is
 -8.46 mm/100 m.
The pressure fall for 10.78×100 meters is -91.20 mm.

B = pressure at the cloud base in convection state $(763.27 - 91.20)$... 672.07 mm.
 $(B_0 - B)_\alpha$ = difference of pressure at cloud base 2.37 mm.
 T = temperature (absolute) at base of cloud $(273 + 9.3)$ 282.3°
 w_α = vertical velocity at the top of the α -stage 23.91 m.p.s.

β -stage.

The gradient in the static state is -7.11 mm/100 m.
The gradient in the convection state is -7.40 mm/100 m.
The height is 17.28×100 meters.
 $(B_0 - B)_\beta = (7.40 - 7.11) \times 17.28 = 0.29 \times 17.28 = 5.01$ mm.
 $(B_0 - B)_\alpha = (8.46 - 8.24) \times 10.78 = 0.22 \times 10.78 = 2.37$ mm.
 $(B_0 - B)$ = total change in the pressure 7.38 mm.
 T = temperature at the top of the β -stage 273°
 B = pressure at the top of the β -stage 544 mm.
 w_β by the formula 46.11 m.p.s.

It is seen from the preceding computation that we have found a vertical velocity of

$w_\alpha = 23.91$ meters per second at the top of the α -stage, and
 $w_\beta = 46.11$ meters per second at the top of the β -stage.
 w_δ = small. The velocity is evidently small at the top of the cloud.

There are, however, a number of reasons for thinking that these large values of w are erroneous, and must be greatly diminished. Since the discussion may be of interest, altho no satisfactory decision is reached as the result of it, the following circumstances should be considered.

(1) The pressures as computed for the cloud levels have been directly compared with the static pressures as determined from the gradients derived from the Barometry Report, and therefore any error in either of these steps must be allowed for in the comparison. It is noted that while there are no obvious errors in the work, we are yet dealing with small quantities,

$$(B_0 - B)_\alpha = 2.37 \text{ mm.} = 0.093 \text{ inch, and}$$

$$(B_0 - B)_\beta = 5.01 \text{ mm.} = 0.197 \text{ inch,}$$

and that it will require great precision in the meteorological data to make these figures perfectly reliable.

(2) It has been practically assumed that these two types of pressure are in action simultaneously on the same plane within the cloud itself. As a matter of fact the congestive circulations producing a cumulo-nimbus cloud are very complex, and they involve masses of air outside the cloud limit.

In the case of the Cottage City waterspout the cold air of the anticyclone flows over the ocean strata, and immediately sets up a series of currents of which the cloud itself is one effect. This indicates that the normal static pressure has already been disturbed, and, therefore, the vertical current begins to move before such wide variations in $(B_0 - B)$ as 2.37 or 5.11 mm. can occur. In fact the current tends to fill up the pressure difference as soon as it begins to diverge from the normal state, and if it were possible to trace this variation exactly, or if, conversely, we could accurately measure the

velocity at the several points, then the problem could be finally resolved. Unfortunately neither of these measurements can be made and therefore we are limited to general discussions. It is my impression, however, that these facts indicate that the pressure difference will seldom be as large as 1 millimeter or 0.040 inch, which implies a vertical velocity of only 15 meters per second. *There are reasons for thinking that this is the maximum value of the vertical velocity, and that it seldom can occur in nature.* Probably the vertical velocity is usually something like 5 meters per second or even less, in the midst of such a cloud, but this is merely an opinion.

(3) It is very probable that the vertical velocity increases to a maximum within the cloud at about the level of the top of the β -stage, and that it is small at the top of the cloud, also at the sea level except in the midst of the vortex. The appearance of the cloud, as usually observed, shows that there is a rapid vertical growth in the central mass from the base upward, and that a sort of boiling with overflow to the sides takes place, except as disturbed by penetrating into other moving strata. Since several theories of hail formation depend upon a very strong vertical current, it has been important to point out the fact that the vertical velocity does not probably exceed 15 meters per second, and that evidence implies that this is generally too high. We will consider briefly the theories proposed for the formation of large hailstones, and then make such suggestions as seem warranted by the conditions determined in this special cloud.

APPROXIMATE POSITION OF THE ISOTHERMS AND ISOBARS IN THE COTTAGE CITY WATERSPOUT CLOUD.

The practical difficulty of solving this problem lies in the fact that the data are wanting with which to compute the value of ΔB in the formula,

$$w^2 = 574.06 \frac{T}{B} \Delta B.$$

It is important to approach the true value at least approximately, if possible, and for that purpose I have made the following trial computations, shown in Table 67. The data for the B , t , $R.H.$ as selected for the three hours, 4 p. m., 1 p. m., and 10 a. m., are indicated in English measures, and transformed into metric measures. The mean temperatures, θ , of the air column in the α , β , γ , δ stages are determined as follows: Plot the values of t , 58° , 67.5° and 65° on the sea level; compute from the gradients of Table 51, summary of data, the heights at which 58° and 65° occur over the 1 p. m. and 10 a. m. columns, and draw the isotherms; plot the temperature 48.7° at the height of the bottom of the β -stage, 32° at bottom and top of the γ -stage, and 10.4° at top of the cloud, as indicated on fig. 39. Then, I have drawn the isothermal slopes by judgment, admitting that they may need modification to be true to nature, tho there is no criterion now available. The temperature was determined at each 500-foot level in the three columns, and for the intermediate 250-foot points by taking the means of successive pairs of values. This gives 6 pairs in the α -stage, 12 in the β -stage, and 13 in the δ -stage. The mean of the several α , β , γ , δ groups gives the values of θ for the several stages as shown in Table 67. The values of the vapor tension, e , were computed, taking the relative humidity, $R.H.$, given for the α -stage, and 100 per cent in the other stages. They may not be exactly correct, but they are sufficient for a close barometric reduction by the formula,

$$\log B = \log B_0 - m + \beta m + \gamma m,$$

the γm term being negligible in the latitude of Cottage City.

For the purpose of comparing the resulting pressures for the several stages found by the static formula just given and those found by the thermodynamic formulas as given in Table 51, summary of data, we select for 1 p. m. the following figures:

Stages.	Static method. mm.	Thermodynamic method. mm.
B_δ (top)	414.56	414.50
B_γ	539.02	539.00
B_β	544.02	544.00
B_α	672.30	672.00
B_a (bottom)	763.27	763.27

This shows that these two groups of formulas and the dependent tables work together in entire harmony, considering the very different ways of using the temperature terms, since they are involved by means of their diverse functions. The resulting isobars for the stages are indicated on fig. 39, assumed approximate position of the isotherms and isobars. It is noted that from the beginning to the end of the disturbance, from 10 a. m. to 4 p. m., the isobars in the cloud region change by about 1 mm. of mercury. In the immediate neighborhood of the waterspout, 1 p. m., there may have been a series of abrupt rises and falls of the pressure, such as are usually found on the barograph traces in thunderstorms; the extent of these I can not determine for this case, but the important fact is that the range of ΔB must have been about 1 mm. in the cloud, because it is hardly probable that it should have exceeded for any short interval the total change between the 10 a. m. and 4 p. m. extremes. By entering 1 mm. in the formula, we find

$$w = 15.52 \text{ meters per second.}$$

FIG. 39.—Approximate position of isotherms and isobars.

This agrees with my previous estimate of the vertical velocity. It is evident that in the other computation, where the mean gradient for the month was used, as derived from the Barometry Report, namely, -8.24 for the α -stage and -7.11 for the β -stage, Table 51, we assumed that the conditions for the waterspout were the same as those of the mean of August

TABLE 67.—*Computation of the isobars at three hours, August 19, 1896.*

4 p. m.	1 p. m.	10 a. m.
$B=30.10 = 764.54 \text{ mm.}$ $t=58.0^\circ = 14.44 \text{ C.}$ $R. H.=60\% \quad e=7.33 \text{ mm.}$ $\theta_a=51.4 \text{ F.}=10.78 \text{ C.}$ $\theta_\beta=36.7 \text{ F.}=2.61 \text{ C.}$ $\theta_\delta=19.5 \text{ F.}=-6.94 \text{ C.}$	$B=30.05 = 763.27 \text{ mm.}$ $t=67.5^\circ = 19.72 \text{ C.}$ $R. H.=64\% \quad e=10.92 \text{ mm.}$ $\theta_a=58.8 \text{ F.}=14.89 \text{ C.}$ $\theta_\beta=39.9 \text{ F.}=4.39 \text{ C.}$ $\theta_\delta=21.5 \text{ F.}=-5.8 \text{ C.}$	$B=30.03 = 762.76 \text{ mm.}$ $t=65.0^\circ = 18.33 \text{ C.}$ $R. H.=67\% \quad e=10.49 \text{ mm.}$ $\theta_a=57.0 \text{ F.}=13.89 \text{ C.}$ $\theta_\beta=41.7 \text{ F.}=5.39 \text{ C.}$ $\theta_\delta=23.9 \text{ F.}=-4.50 \text{ C.}$
$(a) \ H=1078 \quad \log B_0=2.88340$ $\theta=10.78 \quad m=-.05625$ $e=7.33 \quad \beta m=+18$ <hr/> $\log B=2.82733$ $B=671.94$	$H=1078 \quad \log B_0=2.88268$ $\theta=14.89 \quad m=-.05546$ $e=10.92 \quad \beta m=+28$ <hr/> $\log B=2.82750$ $B=672.30$	$H=1078 \quad \log B_0=2.88239$ $\theta=13.89 \quad m=-.05561$ $e=10.49 \quad \beta m=+26$ <hr/> $\log B=2.82704$ $B=671.49$
$(\beta) \ H=1728 \quad \log B_0=2.82733$ $\theta=2.61 \quad m=-.09286$ $t=7.22 \quad \beta m=+32$ $e=7.58$ <hr/> $\log B=2.73479$ $B=542.99$	$H=1728 \quad \log B_0=2.82750$ $\theta=4.39 \quad m=-.09226$ $t=9.44 \quad \beta m=+38$ $e=8.80$ <hr/> $\log B=2.73562$ $B=544.02$	$H=1728 \quad \log B_0=2.82704$ $\theta=5.39 \quad m=-.09193$ $t=10.17 \quad \beta m=+39$ $e=9.24$ <hr/> $\log B=2.73550$ $B=543.88$
$(\gamma) \ H=74 \quad \Delta B=-5.00$ <hr/> $B=537.99$	$\Delta B=-5.00$ <hr/> $B=539.02$	$\Delta B=-5.00$ <hr/> $B=538.88$
$(\delta) \ H=2062 \quad \log B_0=2.73077$ $\theta=-6.94 \quad m=-.11477$ $e=4.57 \quad \beta m=+28$ <hr/> $\log B=2.61628$ $B=413.31$	$H=2062 \quad \log B_0=2.73161$ $\theta=5.83 \quad m=-.11430$ $e=4.57 \quad \beta m=+28$ <hr/> $\log B=2.61759$ $B=414.56$	$H=2062 \quad \log B_0=2.73149$ $\theta=-4.50 \quad m=-.11373$ $e=4.57 \quad \beta m=+28$ <hr/> $\log B=2.61804$ $B=414.99$

taken day and night. But the waterspout occurred at midday, and the gradient was probably nearer the adiabatic rate, -8.46 for the α -stage and -7.40 for the β -stage, than was supposed. The practical difficulty of successfully treating this part of the discussion is a great barrier to concluding the analysis of the dynamic conditions as derived from the thermodynamic state of the cloud, and additional observational data are much needed under similar thunderstorm actions. Finally, I adopt as the most probable maximum velocity of the vertical current

$$w = 15 \text{ meters per second.}$$

THE BUILDING OF HAIL.

The literature of the discussion of the many problems connected with the making of hailstones in the air is very extensive, but the following references are sufficient to place the subject before the reader in its most recent phases:

Recent Advances in Meteorology, William Ferrel. Appendix 71, Annual Report of the Chief Signal Officer for 1885, Part 2. Pp. 302-315.

Lehrbuch der Meteorologie, J. Hann. 1901. Pp. 682-699.

Die Bildung des Hagels, Wilh. Trabert. Meteorol. Zeitschr., October, 1899. Pp. 433-447.

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Doctor Trabert's paper contains many references to other papers on hail.

The physical structure of hailstones is about as follows:

1. *Central nucleus of opaque snow or snowy ice*, consisting of snowflakes and ice crystals mixed with air bubbles from 0.1 to 0.5 inch in diameter.

2. *Layer of clear ice*, or pellucid material containing incised air cells and liquid in radiating inclosures. It is 0.1 to 0.2 inch thick and terminates in a sharply defined spherical boundary, or else in a more irregular boundary to which adhere mammillary masses of soft snow, which may be 0.1 inch in thickness. The clear layer itself is built up of a collection of many small drops, which are instantly stiffened, as in *undercooled water*, which is 5° to 10° below the freezing temperature, when water falls below zero without freezing and then sets with a shock. The ice crystals are mixed in a motley array.

3. *A series of opaque and clear layers succeed each other*, as

many as fourteen having been counted, the last layer usually being opaque and adhering to a very thin layer of clear ice.

The diameters of hailstones vary from 0.5 inch to 4 inches; a frequent size is 1 inch to 2 inches. The shapes of hailstones may be divided into two classes: (1) Those which have a thick base and pointed top, such as conical with a flat base, pyramidal with a round base, pear shape with a concave base, and mushroom shape with the table downward. In these the accretions are chiefly on the lower side of the body, as if gained in falling thru layers of snow and water drops to the ground. (2) Those which are regularly disposed about a center, as if they grew regularly from all sides, and are spherical, ellipsoidal, lens-form and hemispherical, the ellipsoidal being most common. The stones frequently indicate some effects as from a rotary motion about an axis, so that they grow in a certain plane more readily than in other places. The clear ice forms chiefly on the large plane like a tooth-shaped disk, the forms being many and irregular, depending on the accumulation of hexagonal ice crystals.

The order of construction from the center outward, if there were no repetition of the layers, would probably be:

1. Center = snow belonging to the δ -stage in the cloud.

2. Snow and ice mixed = the undercooled crystals of the γ -stage.

3. Clear ice = the gradual cooling ice of the β -stage in contact with a cold nucleus.

If there are repetitions, it follows that these stages are mixed in the internal circulation of the cloud, and that they are brought in contact with the nucleus in succession by falling, or by some other mechanical process. The undercooling of the γ -stage and δ -stage must be due to sudden transitions of the stone from one layer to another having different temperatures. The gradual cooling must be due to the contact of saturated water drops in the β -stage with the nucleus which has passed out of the δ -stage and the γ -stage into the β -stage. The irregularities merely record the congested state of the air and a mixed condition of the δ - γ - β -stages.

Hailstones are usually of about one kind in the same storm, but they change their type from storm to storm. Hail is formed at the rear of the rising column of warm air, at the place of marked changes in the isotherms, when the barometer is beginning to rise rapidly and the wind shifts from the south to the northwest. This is the locus of the contact of two counter currents of air having very different temperatures, and hail formation is one of the results of the rapid progress of the warm and cold layers toward thermal equilibrium. The energy difference, which marks the departure from the normal equilibrium, does not lie in a vertical direction so much as in a horizontal direction. There is a rapid rise of warm air with condensation of the vapor, as in a thunderstorm, and the lightning usually occurs at about the time of hailfall, but it is sometimes earlier and at other times later, and not necessarily simultaneous. On mountains it is said that there is always lightning with hail and in the valleys sometimes lightning with hail. On mountains there are observed to occur simultaneously lightning, undercooled drops, and snow crystals. In falling thru the warm α -stage the outer opaque coating of the hailstone becomes covered with liquid water and in this condition the stone falls to the ground.

THEORIES OF THE FORMATION OF HAILSTONES.

There are many theories regarding the mode of formation of hailstones, in each of which there is probably an element of truth. None of them can be said to be entirely satisfactory, and yet it is very likely that nearly all of the assigned natural causes and effects generally operate in producing the phenomena. The two principal facts to be accounted for are, (1) the presence of the cold which causes the sudden stiffening of the water drop at undercooled temperatures, and (2) the alterna-

tion of snow and water materials in the successive layers. In the sudden cooling of the water drops there is evolved a considerable amount of latent heat, and the cold must be present to such an extent as to overcome the restraining effect of this latent heat, and yet produce cooling as by a shock of the molecular material in the water. The theories may be briefly summarized as follows:

1. *The oscillation theory.*—It is assumed that two cloud layers of different temperatures are superposed, and that a hailstone oscillates up and down between them under an electrical attraction and repulsion, one cloud being charged with negative electricity, and the other with positive electricity. This theory is now considered unnatural and arbitrary, and it certainly is not true, because no electrical forces exist in clouds capable of thus moving heavy stones up and down in the presence of gravity.

2. *The orbital theory.*—Professor Ferrel postulated a vertical orbit in the cloud, in connection with an internal vortex tube having a vertical axis, and supposed that the stones past around this thru considerable changes in altitude, and thru masses of different structure. Such a flow of air inside the cloud is very improbable, and there is no evidence that the cloud thus rotates. A modification of this view is found in the horizontal roll which very frequently exists on the back side of the warm ascending current, at the place of the most active mixing with the cold column. It is very likely that this does often develop in thunderstorm clouds, and indeed, there may be several such rolls on horizontal axes, and their action may well produce certain effects upon the construction of hailstones of different types; the effects are confined to merely differential variations of the typical structures. The vertical component can hardly lift the stones, except those of the smaller sizes. If the upward current on one side retards a freely falling stone, on the other side of the roll it would accelerate its fall, and so discharge it from the local action in the cloud by this impulse.

3. *The upward current theory.*—The sustaining force of a strong upward current of air in the midst of the cloud, whereby a hailstone is held aloft for a considerable time while it receives accretions from the contents of the ascending stream, acting especially on the under side, is, doubtless, the most important theory to be examined. The growth on the underside of a hailstone can be accounted for either by falling from a considerable height thru the cloud, or by being sustained at a given height by an upward flowing current. There are several difficulties if not objections to this theory, when taken as the single cause of the formation.

(1) The condensation products carried in the vertical current do not seem sufficient to produce the largest stones.

Let W = grams of water in 1 cu. meter.

$$\frac{W}{10^6} = \text{grams of water in 1 cu. centimeter.}$$

A stone of section πr^2 falling thru a height dh will gain $\frac{W}{10^6} \cdot \pi r^2 \cdot dh$ grams, which is equal to a volume increase of

$$4\pi r^2 \cdot dr. \text{ Hence, } dr = \frac{W}{4 \times 10^6} dh, \text{ and } r_2 - r_1 = \frac{W}{4\,000\,000} \times$$

$(h_2 - h_1)$. If the height of fall is 2 kilometers (200,000 cm.), and $W = 4$ grams, at freezing temperatures, then $\Delta r = 0.2$ cm. = 2 mm. This is Trabert's argument, and he thinks it does not fully account for the large stones which are found weighing as much as 250 to 1000 grams. This view conceives the stones to form in the δ and γ -stage as ordinarily stratified in a quiet cloud, but I think that suitable modification can be indicated, which will to some extent avoid the difficulty.

(2) The stream lines around the stone will doubtless carry off some of the particles of water without their touching the

stone itself, and this will tend to diminish the quantity that is actually deposited, thus strengthening the former objection that the total quantity of deposit is insufficient to produce the mass of the observed hailstones.

(3) It is not easy to account for the concentric layers on the upward current theory, or the downward fall theory in a simple cloud.

(4) In seeking to maintain this current theory of accretion, Professor Schreiber, it seems to me, has assumed excessive heights and improbable velocities in the ascending currents. Thus, he makes two assumptions: (a) that the vertical velocity increases steadily, at the rate of 3 meters per second per 1000 meters of altitude, as in the second column of the following table; (b) that the vertical velocity increases at the rate of 7.5 meters per second per 1000 meters, up to 20,000 meters of altitude, and then diminishes at the same rate, down to 0 at 40,000 meters, as in the third column of the following table.

Schreiber's assumed vertical velocities.

Height in meters.	(a)	(b)
	<i>m. p. s.</i>	<i>m. p. s.</i>
30000.....	90	75
20000.....	60	150
10000.....	30	75
5000.....	15	37.5
4000.....	12	30.0
3000.....	9	22.5
2000.....	6	15.0
1000.....	3	7.5
0.....	0	0

He makes two other assumptions for trial, (1) that the vertical current has no limit in height, and the same velocity and density thruout, and (2) that the velocity is the same thruout, but that the density diminishes with the height. He discusses the sorting velocities which separate the stones of different diameters, those larger than the critical velocity falling to the ground, and those smaller rising in the current and growing to larger size in preparation for a fall. It may be remarked, generally, that cloud heights above 10,000 meters are rarely measured, and that the vertical velocities are a maximum within the cloud, probably at the height of the γ -stage, rather than at the top, somewhat as assumed in his fourth trial (p. 62), but by no means at such large values of the current. Schreiber asserts that hail forms at the top of such lofty clouds as 30,000 meters and in vertical currents of 100 meters per second, which it seems to me is impossible in view of the fact that such clouds do not exist, and that by adiabatic laws the γ -stage is seldom higher than 6000 meters in the most favorable summer conditions. In this connection refer to my discussion of the heights of the several stages, Cloud Report, Annual Report Weather Bureau, 1898-1899, pages 720-723, and chart 74. There can be little doubt that we must confine the formation of hail to the region 3000-7000 meters above the ground, and usually to the middle height, most frequently near the 5000-meter level. It is noted that Schreiber assigns a vertical velocity of 15 m. p. s. at the 5000-meter level, and that this agrees with the maximum vertical velocity which it seems probable can be developed in an ordinary summer cloud. This is not strong enough to sustain a hailstone of 1 cm. diameter, which is a small specimen, as a velocity of 20 m. p. s. is required for that purpose, while 40 m. p. s. is required to sustain a large stone 4 cm. in diameter.

4. *The electrical attraction theory.*—In order to escape from such difficulties as those just enumerated in the vertical current theory, Trabert advocates the theory that the sudden accumulation of drops on the nucleus at undercooled temperatures is due to the electrical charging of the nuclei at the instant of a lightning flash, the surface charges having the

power to attract water drops to the charged surface. The drops of a jet of water are thus suddenly drawn together by an electrified piece of wax placed near it. The drops fly together when changes take place in the electric field surrounding them. Some observers say that there is no hail without the electric phenomena. Each layer of ice is made suddenly by electric impulses which follow in succession for the several layers. The deposit of a layer brings the undercooled temperature up to the freezing point. The escaping heat of the undercooled mass in condensing makes a water layer on the outside which changes to ice. There is a considerable quantity of latent heat evolved in the process of water and ice formation. Hail weather and lightning weather are alike in kind and different in intensity. Thunderstorms are associated with the horizontal roll due to overturning, and hailstones with the vertical vortex due to excessive convection currents. It has been suggested that the heat of the convection process is transformed into electricity and that the required cooling is produced in that way, but of this there is little evidence. The cooling is also referred to sudden expansion in the air, but this would produce so great changes in the barometer that it would be readily detected.

This electrical theory ought to play a part in the formation of hail at times, but it is hardly demonstrable that hail does not fall without lightning, and certainly it is not shown that a flash of lightning occurs at the time of the deposit of the several stratified layers. A hailstorm often lasts many minutes, and during that time there must be, on this theory, such an incessant recurrence of lightning to match the numerous layers of ice that it would be a very conspicuous event. There are many instances known in which the lightning seems to have really followed the hail by many seconds, but it should evidently precede it, if the time allowance for the fall from the cloud to the ground is subtracted from the instant when the hail is seen to fall upon the ground. F. W. Very writes:

Severe hailstorms are almost universally accompanied by thunder and lightning, but the electrical display is apt to lag, and even to attain its greatest development as the storm advances to its close.

The rising air carries up the low surface potential on the front side of the hail squall, and the descending air brings down the high potential of the upper levels, so that there is an increase in the difference of potential at the hail level which causes horizontal flashes in the cloud, for the greater part. The earth's negative charge is carried aloft to the hailstones, which are often negatively charged. Snow which forms in the high levels is usually positively charged. Reversals of the ordinary disposition of the electric potential have been noted as the effect of snow, hail, and water inductions brought to the surface of the ground.

5. *The stratification theory.*—After the foregoing examination of the theories that have been heretofore proposed for the explanation of the growth of hailstones, I proceed to examine the subject from a new point of view, which seems to me to offer certain advantages over the other theories, and to embody the best points of them all. This I call the stratification theory. It happens that a hailstorm cloud, which is merely an intense form of thunderstorm cloud, really consists of two component portions separated from each other by isothermal surfaces inclined forward from the vertical. On the front side the air is much warmer than on the back side, and along the line of separation the contour is strongly stratified by the mutual interpenetration from opposite directions of layers having different temperatures.

Fig. 40, "Stratification of the β and δ -stages in a thunderstorm cloud with hail", roughly illustrates the idea. Such storms begin in consequence of the transportation of cold air into a region of warm air, and in many cases the difference of temperature amounts to as much as 20° F. The tendency for

such masses of air at different temperatures is to mix intimately and irregularly in order to restore the thermal equilibrium as rapidly as possible. The cold air is carried forward in the high levels, and like a sheet overflows the warmer lower layers, as is indicated by the first formation of clouds of the cirrus type, which later change into alto-cumulus and alto-stratus types.

FIG. 40.—Stratification of β - and δ -stage in a cloud with hail.

The body of warm air tends to rise and interpenetrate the cold air in a congested circulation including numerous minor whirls and small vortices. On the western side of the column of rising warm air the tendency to stratification of the warm and cold layers in horizontal directions is very pronounced, the sheets of different temperatures penetrating strongly at a series of intervals in elevation, so that they lie over each other on a given vertical in succession which may be repeated many times. The boundary between the β -stage and the δ -stage, or the course of the γ -stage, is therefore folded upon itself several times in a vertical direction.

For example we may suppose that the temperatures are arranged in some such manner as the following:

- Let $\beta = +15^\circ \text{C.}$ and $\delta = -2^\circ \text{C.}$ in the lowest fold;
- let $\beta = +10^\circ \text{C.}$ and $\delta = -4^\circ \text{C.}$ in the second fold;
- let $\beta = +5^\circ \text{C.}$ and $\delta = -6^\circ \text{C.}$ in the third fold;
- let $\beta = +0^\circ \text{C.}$ and $\delta = -8^\circ \text{C.}$ in the fourth fold;

The temperatures in the β -stage fall off more rapidly than in the δ -stage, and the difference between them diminishes with the height.

The snow nucleus, starting from a great height, meets the water carried aloft in the warm strata, is coated with the

drops, which are chilled by its lower temperature and frozen in irregular semicrystalline forms. The vertical current at even moderate velocities is able to carry up all the water contents in the form of drops, and they are injected as it were sideways from a fountain into the higher strata. The snow nucleus is therefore simply exposed to a spray of water drops, brought from the lower strata where high vapor contents prevail, because of the warm air occupying the lower levels before they were disturbed by the overflowing anticyclonic cold. The cold nucleus, therefore, suddenly condenses a layer of clear ice, or ice and snow when mixt by the minor vortices and horizontal rolling of the air. The small hailstone then falls by gravity thru successive stratifications of the snow and rain stages, it grows on the underside by special accumulations there, and finally reaches the ground, having received as many layers as there are distinct horizontal minor stratifications. The undercooling takes place chiefly in the highest stratifications, and ice or snow crystals are found deposited in the inner layers of the hailstone. The under cooling diminishes with the descent so that the outer layers are watery or simply opaque.

There evidently exists a series of small horizontal rolls produced by the dynamic action of the interflowing sheets, where the mixture of air at different temperatures is facilitated by drawing it out into thin ribbons, as in ordinary cyclonic circulations. The lowest cold stratum flows forward on the ground, producing the squall of cold air that precedes the rainfall. An examination of the isotherms and isobars on fig. 39 shows that this distribution of the air currents is the probable one, allowing for the minor configurations on the edges of the mixing masses. The isobars show that at the sea level the air flows forward, but in the upper levels it flows backward at the time of the hailstorm. The isotherms show that there is an excess of upward velocity at the line of separation, and also that the flow is backward in the higher levels. The production of lightning discharges under these conditions, especially in the region where the cloud is serrated as to temperatures, is evidently to be anticipated, in consequence of the rapid changes occurring in the thermal conditions and the water contents. The hailstones may therefore be heavily charged with positive electricity, or even with negative electricity, under these circumstances, and the fallen hailstones may exhibit electrical states by no means uniform from storm to storm.

It is desirable that numerous computations be made on the data that may be obtained from the surface observations in thunderstorms and in hailstorms, with the view of transforming our inferences regarding the thermal operations going on in the midst of such clouds into more definite knowledge. The formulas and the tables employed in this paper are satisfactory, and it is possible to accomplish much by using only our surface observations. It is, however, very important to supplement such studies with the actual observations in the clouds by balloons and kites.

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